

Fracture processes in quasi-brittle materials: Linking heterogeneity to crack roughness

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ABSTRACT

The roughness of crack surfaces of heterogeneous quasi-brittle materials such as concrete, bones, rock and wood is linked to the spatial extension of the fracture process zone. Modelling of this zone is required to predict the influence of size on the nominal strength of geometrical similar structures made of quasi-brittle materials or crack spacing in reinforced structures. The aim of this study is to further investigate the link between the heterogeneity of the material and the roughness of crack surface by means of discrete network analyses of the failure process of heterogeneous quasi-brittle materials. In the present study, a 3D periodic unit cell of an irregular structural network representing a heterogeneous material is subjected to direct tension. The material meso-structure is modelled by a random field of the material strength mapped on the irregular network of discrete elements. The influence of the standard deviation of the random fields on the crack roughness is investigated.

Key Words: *Fracture; roughness; discrete; dissipation; heterogeneous material*

1. Introduction

The fracture process in specimens made of heterogeneous quasi-brittle materials such as concrete, bones, rock and wood, subjected to direct tension is characterised by a transition from distributed micro-cracking to a localised stress-free macroscopic crack separating the specimen, whereby the final stress-free crack surface is rough. In [1], it was aimed to develop a procedure to link this roughness of the stress-free crack surface to the width of the fracture process zone, which could then be used to calibrate the interaction radius of nonlocal constitutive models for predicting the failure process of structures made of these heterogeneous materials (Figure 1). The key of this approach was to determine a measure from the roughness, which could be linked to the distribution of dissipated energy obtained from models to be calibrated. This measure is the standard deviation of the height of the crack surface from the mean crack plane. One of the assumptions of this approach was that the large majority of fracture energy was dissipated by the localised crack and not the distributed micro-cracking. This was supported by 2D numerical structural network analyses of specimens with random material strength properties subjected to direct tension, and experimental results reported in the literature [2]. In [1], the new calibration approach was applied to the fracture of a concrete beam subjected to three-point bending, which contained low strength aggregates. The roughness of the crack surface measured in these experiments was small, which was explained by low strength inclusions, which fractured during the tests. In the present study, the roughness of crack surfaces obtained from direct tensile tests is further investigated by means of a numerical approach based on a structural network model. The new contribution is the extension of the previous 2D modelling approach to 3D and an investigation of the influence of the standard deviation of the random material strength on the roughness.

2. Method

The method to computationally determine the roughness of crack surfaces of direct tensile tests is presented here by describing the discretisation approach, constitutive model, random field and roughness determination.

The spatial discretisation of the direct tensile specimens is performed using a 3D structural network model described in [3], combined with a 3D periodic cell proposed in [4] (Figure 2). The structural

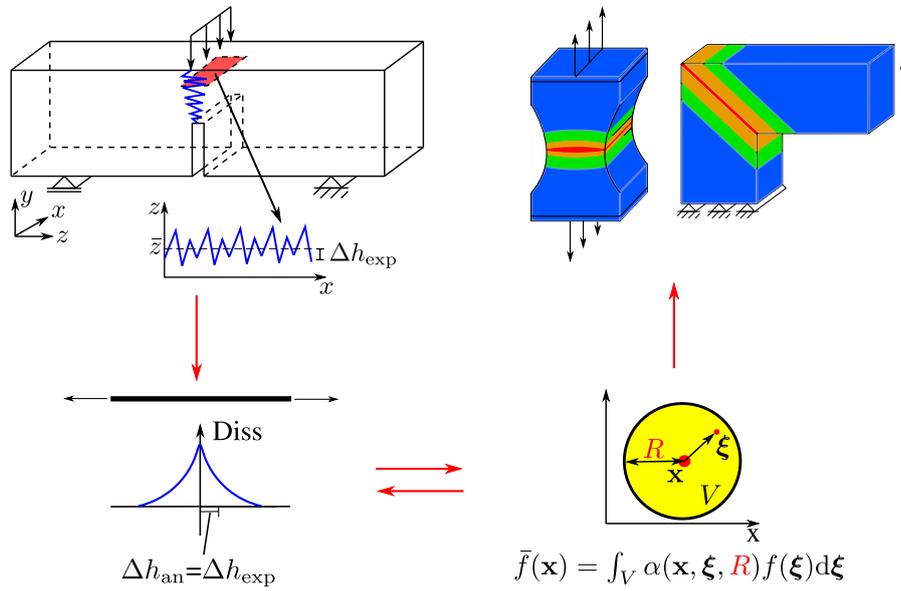


Figure 1: Schematic overview of the calibration strategy of the interaction radius of nonlocal constitutive models: Roughness measurements from experiments or analyses (left top) are used to determine the distribution of dissipated energy (left bottom). This distribution is used to calibrate the nonlocal constitutive model (right bottom) which is then used in structural analysis. Adapted from [1].

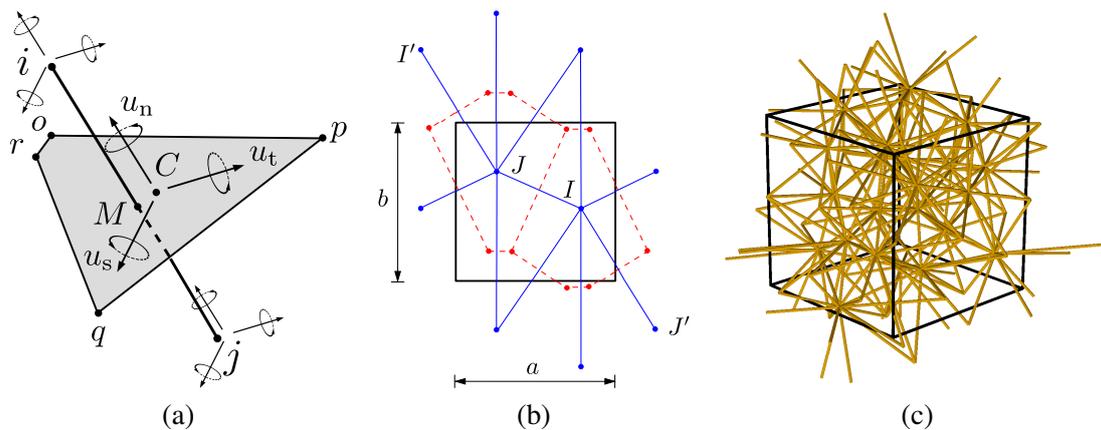


Figure 2: 3D discrete network model: (a) 3D network element, (b) schematic periodic network in the cell with elements crossing the boundary, and (c) 3D periodic cell of discrete elements.

network element in Figure 2a has three translational and three rotational degrees of freedom at each node. The spatial arrangement of the network is based on a Delaunay and Voronoi tessellation of a periodically arranged set of random points following the approach described in [4]. The vertices of the Delaunay tetrahedra are used as nodes for the structural network elements, which are placed on the edges of the tetrahedra. The mid cross-sections of the network elements are the facets of the Voronoi polyhedra associated with edges of the Delaunay tetrahedra [3]. The average element length is controlled by the minimum spacing d_{\min} of the randomly placed points used for the tessellations.

The degrees of freedom of the nodes of an element are linked by kinematic relationships to displacement jumps at point C in Figure 2a, which enters the scalar damage model. The input parameters are the modulus of elasticity E , the strain at tensile strength $\varepsilon_0 = f_t/E$ and the displacement threshold w_f controlling the softening described by the damage model [1]. The strength envelope is an ellipsoid determined by the tensile strength f_t , the compressive strength $f_c = cf_t$ and the shear strength $f_q = qf_t$.

The heterogeneity of the material is represented by an autocorrelated Gaussian random field of the strain threshold ε_0 . The input parameters of the random field are the autocorrelation length l_a and the mean $\bar{\varepsilon}_0$ and a coefficient of variation c_v of the Gaussian distribution. The values of the random field are mapped onto the structural network. The autocorrelation length l_a is chosen to be greater than the spacing d_{\min} used for the network generation.

The roughness of crack surfaces from the analyses is determined from the weighted average and standard deviation of the dissipated energy as described in [1]. Firstly, the dissipated energy and the heights (in the direction of loading) of mid-cross-sections are determined. Then, the weighted mean \bar{z} and the standard deviation Δh of all heights is calculated.

3. Analyses and Results

The influence of the coefficient of variation of the random field of strength used for the direct tensile specimen on the surface roughness was investigated. The structural network for a periodic cell of edge length $h = 0.1$ m was discretised using $d_{\min} = 3$ mm. For the random field, five coefficient of variation values $c_v = 0.05, 0.1, 0.2, 0.4$ and 0.6 were selected. Furthermore, the autocorrelation length was chosen as $l_a = 4$ mm. The mean values of the input parameters are $E = 30$ MPa, $\bar{\varepsilon}_0 = 0.0001$ and $w_f = 0.00004$, $c = 10$ and $q = 2$. For each c_v , 20 analyses with different networks and random fields were performed. The periodic cell was subjected to an incrementally applied axial displacement of 0.2 mm, while keeping the average lateral stresses equal to zero. The axial stress-displacement curve and crack patterns at different stages are shown in Figure 3 for one random analysis with $c_v = 0.2$. The crack patterns are visualised in the form of orange mid-cross-section at which energy is dissipated at this stage of analysis.

Initially, energy is dissipated throughout the specimen. However, very soon after the peak, the zone in which energy is dissipated shrinks and the fracture process zone is mainly formed by one rough crack, which dissipates the majority of energy. This evolution of the fracture process zone is very similar to the 2D results in [1]. According to the present simulations, the width of the fracture process zone is mainly controlled by the roughness of the crack. It should be noted that because of the use of a periodic cell, the location of this crack plane within the specimen depends only on the specific realisation of the random field, and not the specimen boundaries.

The influence of the coefficient of variation c_v on the roughness of the crack was investigated by evaluation the weighted standard deviation Δh of the mid-cross-sections. The preliminary analyses so far, show that an increase of c_v results in an increase of the roughness Δh . The preliminary results indicate that stronger inclusions represented by larger standard deviations result in rougher cracks.

4. Conclusions

3D structural network analyses of direct tension of a heterogeneous quasi-brittle material have shown that the energy during the fracture process was mainly dissipated in a localised rough crack plane. Therefore, the width of the fracture process was determined by the roughness of the crack plane. An increase of the

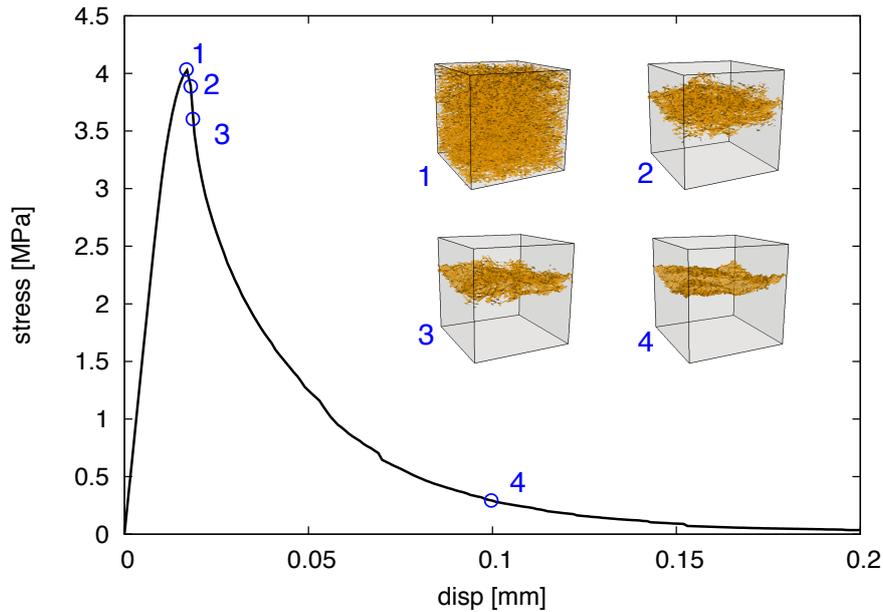


Figure 3: Periodic cell subjected to direct tension: Stress versus displacement and crack patterns (orange mid-cross-sections) at four stages of analysis.

coefficient of variation of the random field of tensile strength resulted in an increase of the roughness of the crack plane.

Acknowledgements

The numerical analyses were performed with the nonlinear analyses program OOFEM [5] extended by the present authors.

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