

# Acceleration techniques for nonlinear finite element analysis of quasi-brittle materials

W. Alnaas<sup>1</sup> and A. Jefferson<sup>1</sup>

<sup>1</sup>School of Engineering, University of Cardiff, Queen's Buildings, Cardiff, CF24 3AA

AlnaasWF1@Cardiff.ac.uk

## ABSTRACT

The nonlinear systems of equations resulting from the finite element modelling of quasi-brittle material are most-frequently solved using Newton-based incremental-iterative schemes. However, the poor convergence characteristics and frequent numerical breakdowns of those solutions schemes, when solving problems involving quasi-brittle materials, make the nonlinear finite element analysis of such materials a truly challenging undertaking. Recently, Alnaas and Jefferson [1] proposed an algorithm, named the smooth unloading-reloading (SUR) method, which circumvents these problems by employing a tangent matrix that is always positive definite but which is exact with respect to the nonlinear UR function. The present paper describes a further development of this method in which an acceleration technique is introduced into the solution algorithm. The performance of three alternative acceleration methods is reported in this paper. The main conclusion from the work is that all three acceleration methods result in a reduction in solution time for a range of problems.

*Keywords: Nonlinear FE analysis; quasi-brittle materials; damage, robustness, incremental-iterative*

## 1. Introduction

Numerical difficulties often arise in the finite element simulation of quasi-brittle materials. Such problems are associated with material softening behaviour and the loss of positive definitiveness of the tangent stiffness matrix. These difficulties often manifest as breakdown of the nonlinear incremental-iterative solution process [2].

As a response to these stability and convergence difficulties, researchers have developed solution algorithms that avoid multiple iterations. These methods include the 'Sequentially Linear Approach' (SLA), which was introduced by Rots [3] and improved later by others. The SLA method uses a 'saw-tooth' function to replace the post-peak softening function. Another approach, which avoids using multiple iterations, is the implicit-explicit "IMPL-EX" approach of Oliver et al [4]. In the IMPL-EX method, a projected state variable (e.g. a damage parameter) is used to determine a predicted consistent tangent matrix that is exact for the current increment but for which a correction is made in the subsequent stress-recovery phase.

Recently, Alnaas and Jefferson [1] developed a new approach called smooth unloading-reloading 'SUR' approach to work with an incremental iterative nonlinear FE solution scheme. This approach circumvents stability and convergence problems by employing a tangent matrix that is based on a smoothed unloading-reloading function. This function has a small positive gradient at its intersection with the principal softening evolution function. A key feature of this method is that it always uses a positive definite stiffness matrix. The authors have shown the approach to be numerically robust, reasonably efficient and accurate.

This paper describes three accelerations techniques to improve the convergence properties of the recently developed SUR approach for a finite element damage model, when applied to quasi-brittle fracture problems.

## 2. Smooth unloading-reloading (SUR) method

The SUR approach uses a target function material  $f_s(r_p)$  and a smooth unloading-reloading (SUR) function  $\sigma_p(r_p, r_{eff})$ , as illustrated in Figure 1. As can be seen, the smooth unloading-reloading function has two parts: I) when  $r_{eff} < a_p r_p$ , for which linear unloading-reloading with a slope  $(1-\omega_{pf})E$  is assumed; and II) when  $r_{eff} \geq a_p r_p$ , for which nonlinear unloading-reloading is assumed. The SUR depends on the damage evolution parameter ( $r_p$ ).

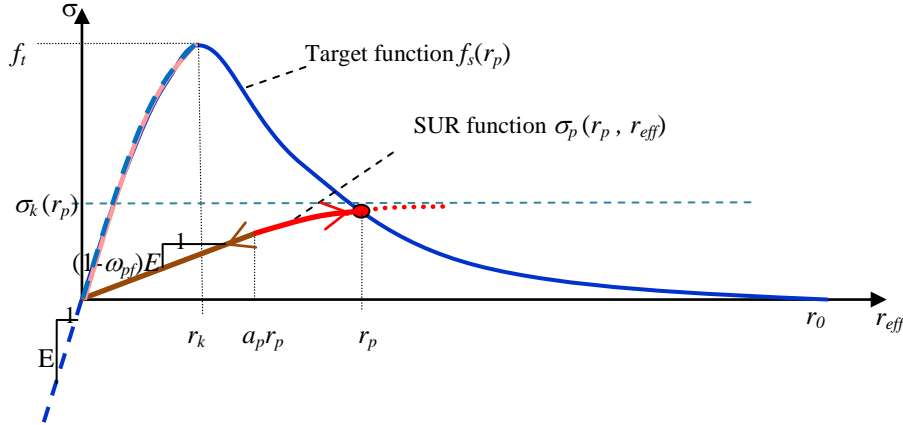


Figure 1: Target and unloading-reloading damage functions.

The target and smooth unloading-reloading functions are given in equations (1) and (2), respectively.

$$f_s(r_p) = \begin{cases} f_t & \forall r_p < r_k \\ f_t \cdot e^{-c_1 \cdot \left( \frac{r_p - r_k}{r_0 - r_k} \right)} & \forall r_p \geq r_k \end{cases} \quad (1)$$

$$\sigma_p(r_p, r_{eff}) = \sigma_k(r_p) \cdot \left[ 1 - \left( 1 - \frac{a_p}{v} \right) \cdot e^{-\left[ \frac{r_{eff} - a_p r_p}{(v - a_p) r_p} \right]} \right] \quad (2)$$

where  $r_{eff}$  is the effective damage parameter,  $E$  is Young's modulus,  $f_t$  is the tensile strength,  $r_0$  is the effective end of the softening curve,  $r_k$  is the damage evolution parameter at the peak of the uniaxial stress curve.  $c_1=5$ ,  $a_p = 0.70$  and  $v=0.75$ . See Ref [1] for more details of the SUR method.

### 3. Acceleration techniques for the SUR method

In this paper, three acceleration techniques are described for improving the convergence performance of the SUR solution procedure. These acceleration techniques are described below.

#### 3.1 Predictive-SUR technique

The concept of the predictive-SUR algorithm relies on two parameters, namely the damage evolution parameter ( $r_p$ ) and the number of iterations ( $it$ ). The predictive function is based on two assumptions: I) The relationship between the number of iterations ( $it$ ) within a time step and the iterative change in the damage evolution parameter ( $\Delta r_{p_i} = r_{p_i} - r_{p_{i-1}}$ ) decays in semi-log space and approaches 0, once stable convergence has been achieved: II) when the slope of ( $it$  vs  $\log(\Delta r_{p_i})$ ) curve starts decreasing, a trial prediction of the damage evolution parameter ( $r_{pp}$ ) can be computed using equation (3). Once the normalised difference between two consecutive predictions e.g. ( $r_{pp_i}$ ) and ( $r_{pp_{i-1}}$ ) is less than 5%, then  $r_p$  is set to the most recently computed trial value, i.e.  $r_p = r_{pp}$ .

$$r_{pp} = r_{pit} + \frac{\Delta r_{p_i}^2}{\Delta r_{p_{i-1}} - \Delta r_{p_i}} \quad (3)$$

### 3.2 Fixing approach

The second acceleration technique named ‘fixing’ is based on a two stage algorithm, in which a damage evolution parameter is updated from the last converged increment in Stage-1 iterations, and then it is fixed in Stage-2 iterations within a step. In this acceleration approach, 3 and 5 iterations in Stage-1 were investigated.

### 3.3 Slack tolerance technique

This technique uses a slightly slacker convergence tolerance at key stages in a computation. The slacker tolerance (1% for the L2 norm of out of balance residual forces) is temporarily triggered when the number of iterations within an increment exceeds a certain limit (e.g. 5 iterations). Subsequently, the convergence tolerance reverts to the standard tighter tolerance of 0.001%.

### 4.1 One-dimensional example

The one-dimensional bar shown in Figure 2 was used. The bar was divided to three linear elements of equal length, with the middle element being assigned a small amount of initial damage such that damage only occurred in this central element. A prescribed displacement 0.2 was applied evenly over 40 and 100 increments in the analysis. The material properties of the bar are:  $E=20\text{GPa}$ , Poisson’s ratio ( $\nu=0.2$ ),  $f_t=2.5\text{ MPa}$ , fracture energy ( $G_f=0.10\text{ N/mm}$ ) and tolerance value for out-of balance force and displacement norms is  $\Psi=0.0001\%$ .

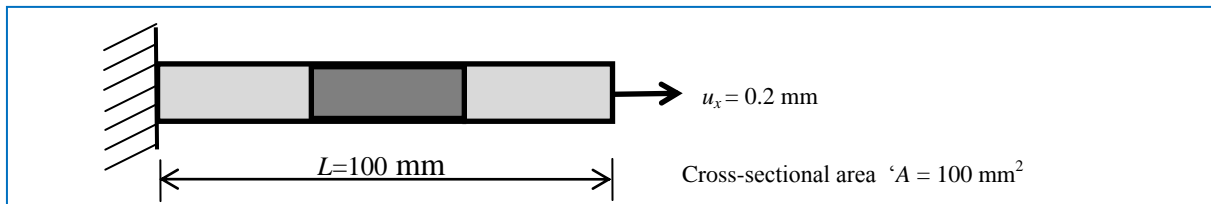


Figure 2: 1-D bar example

The resulting stress-displacement responses from the various analyses are indistinguishable from each other, as can be seen in Figure 3a.

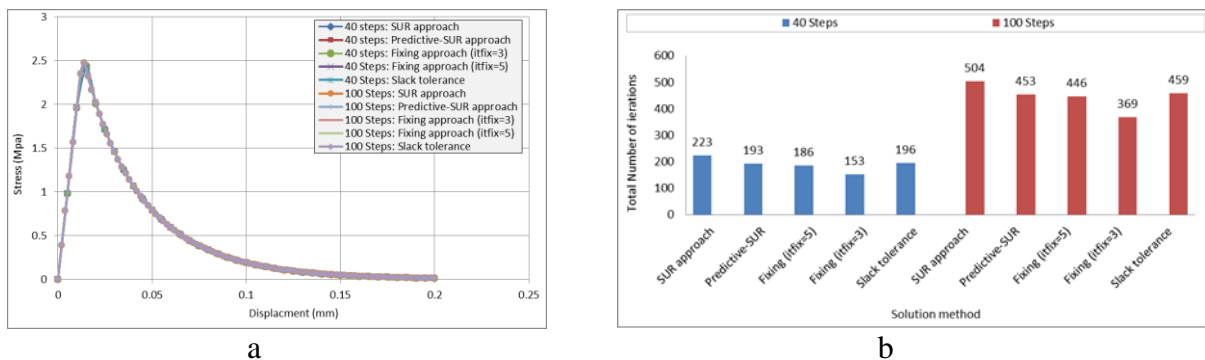


Figure 3: (a) Stress-displacement response, (b) total number of iterations that needed for each solution.

In all sets of analyses, results showed that the three acceleration techniques achieved converged solutions in fewer iterations than the standard SUR solution, see Figure 3b. Furthermore, the ‘fixing algorithm’, with 3 iterations in Stage-1, was on average a little more efficient than the others, as can be noticed in Figure 3b.

## 4.2 Two-dimensional double notched example

The second example is a 2-D double notched specimen subjected to mixed mode loadings by prescribed displacement, as shown in Figure 4a. The analysis was undertaken using 40 and 100 prescribed displacement increments. The material properties of the specimen are:  $E=20$  GPa,  $\nu=0.2$ ,  $f_t=2.5$  MPa,  $G_f=0.10$  N/mm and  $\Psi=0.001$ .

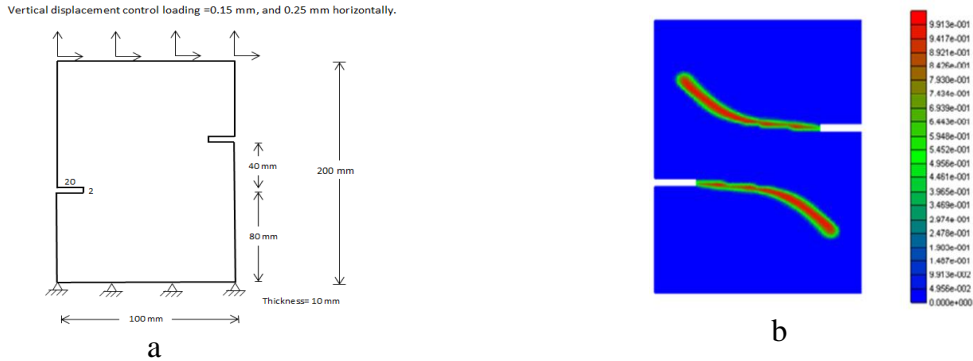


Figure 4: (a) Dimensions of the 2D-double notched specimen, (b) Damage contour plot

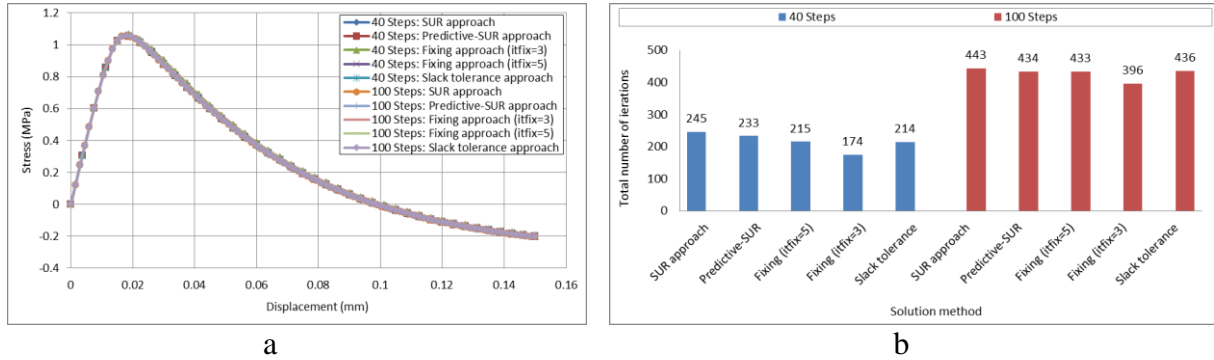


Figure 5: (a) Stress-displacement response, (b) total number of iterations that needed for each solution.

As you can see in Figure 5b, the three acceleration approaches require fewer iterations to achieve convergence, relative to the standard SUR method. Also, stress-displacement curves for all solutions are indistinguishable from each other, see Figure 5a.

## Conclusion

The principle finding of this paper is that all three methods require less computer time than the standard SUR method, with no appreciable effect on the accuracy of simulations.

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