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# DEVELOPMENT OF A 1D CARDIOVASCULAR MODEL DURING PREGNANCY

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**ABSTRACT**

The expectant mother undergoes many physiological changes over the course of pregnancy, many of which affect the cardiovascular system. An existing closed loop 1D cardiovascular network model has been modified, which includes the main systemic, pulmonary and cerebral arteries and veins. The enhanced trapezoidal rule method (ETM) is used to solved the 1D blood flow and lumped parameter system. Lagrange multipliers are use to constrain conservation of mass and conservation of total pressure at vessel junctions. The method is implicit and naturally incorporates coupling between 1D vessels and lumped models. The ETM has been compared with benchmark problems and results show excellent agreement with commonly used schemes. The aim of this research is to develop a cardiovascular model which describes various changes seen during pregnancy. A fully closed loop cardiovascular network is being developed which uses; a five element Windkessel model to connect arteries and veins; and 0D lumped models to describe the heart and valves.

***Key Words:*** *Implicit Solver; 1D Blood Flow; Closed-Loop System; Cardiovascular Network; Pregnancy*

## 1. Introduction

During pregnancy the expectant mothers undergoes many physiological changes. The cardiovascular system needs to adapt to allow up to an additional 50% of maternal blood volume, and, increased stroke volume and heart rate. However, even with these effects it is common that the maternal blood pressure actually decreases. This is partly caused by the vascular resistance decreasing by around 20%, and, a number of maternal blood vessels increase in diameter, particularly those close to the womb, such as the ovarian and uterine arteries. This increase in blood volume also leads to lower blood viscosity. Moreover, as the foetus develops, external pressure on the maternal blood vessels will increase, particularly in the abdominal area.

1D blood flow models have been often used to study wave propagating in arteries, and have been extended to veins. Most so called closed loop models use lumped parameter models for the pulmonary circulation such as [3] rather than treating them as 1D vessels. Mynard and Smolich [4] have recently developed a fully closed loop 1D cardiovascular network which include 1D vessels in systemic arteries and veins and pulmonary arteries and veins, together with a heart model.

An improved version of the simplified trapezoidal rule method (STM) [2] has been developed for 1D blood flow which allows intuitive coupling of 1D vessels and lumped parameter models, such as valves, the heart and vascular beds. The method is applied to benchmark problems from [1] and compared with the results therein. The aim of this research is to develop a cardiovascular model which investigates the mechanisms involved during pregnancy.

## 2. Methodology

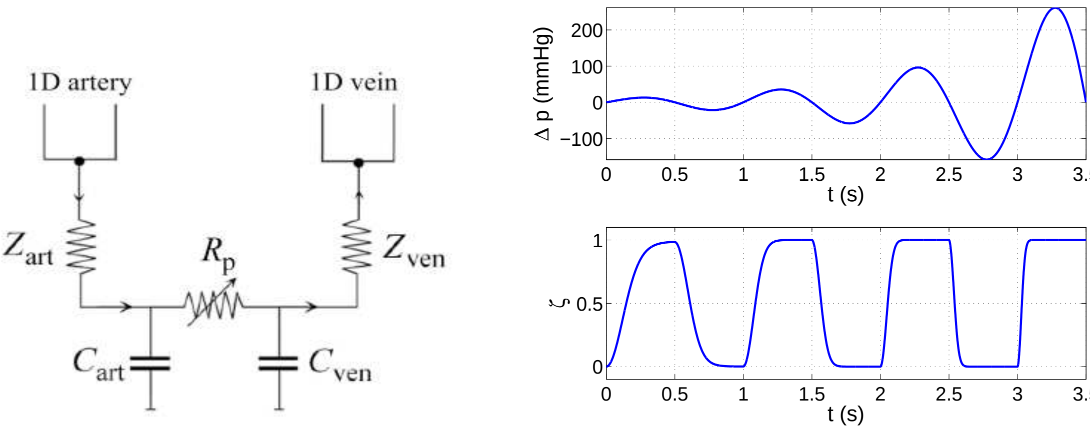
The system of equations that represent the linearised 1D blood flow are the conservation of mass and conservation of momentum

$$C_A^n \frac{\partial P}{\partial t} + \frac{\partial Q^{n+1}}{\partial x} = 0, \tag{1}$$

$$\frac{\rho}{A^n}\frac{\partial Q}{\partial t} + \frac{\rho}{A^n}\left(\frac{\partial \frac{Q^2}{A}}{\partial x}\right)^n + \frac{\partial P^{n+1}}{\partial x} + \left(\frac{8\mu\pi Q}{A^2}\right)^n = 0, \tag{2}$$

where $C_A$ is the compliance; $P$ and $Q$ is the average pressure and average flow rate in the cross-section respectively; $t$ is the time; $A$ is the cross-sectional area; $\rho$ is the density of blood; and $\mu$ is the blood viscosity. The system is closed by the constitutive law linking pressure and area $P(A)$.

The vascular beds are treated as lumped models using five element Windkessel models shown in figure 1(a), such as in [4, 5]. The model contains the characteristic impedance of an artery $Z_{art}$; the characteristic impedance of a vein $Z_{ven}$; a vascular bed resistance $R_p$ and the arterial and venous compliances $C_{art}$ and $C_{ven}$ respectively.




(a) Five element Windkessel model [5]

(b) Valve opening coefficient $\zeta$ for different values of $\Delta P$

Figure 1: Peripheral circulation model and valve opening relationship with pressure difference across valve

The Windkessel model is treated by considering the following two noded resistance and compliance elements, which are coupled together by sharing a pressure node.

$$(P_1 - P_2)\frac{1}{R} = q_1 = -q_2, \qquad C\left(\frac{\partial P_2}{\partial t} - \frac{\partial P_3}{\partial t}\right) = q_3 = -q_4. \tag{3}$$

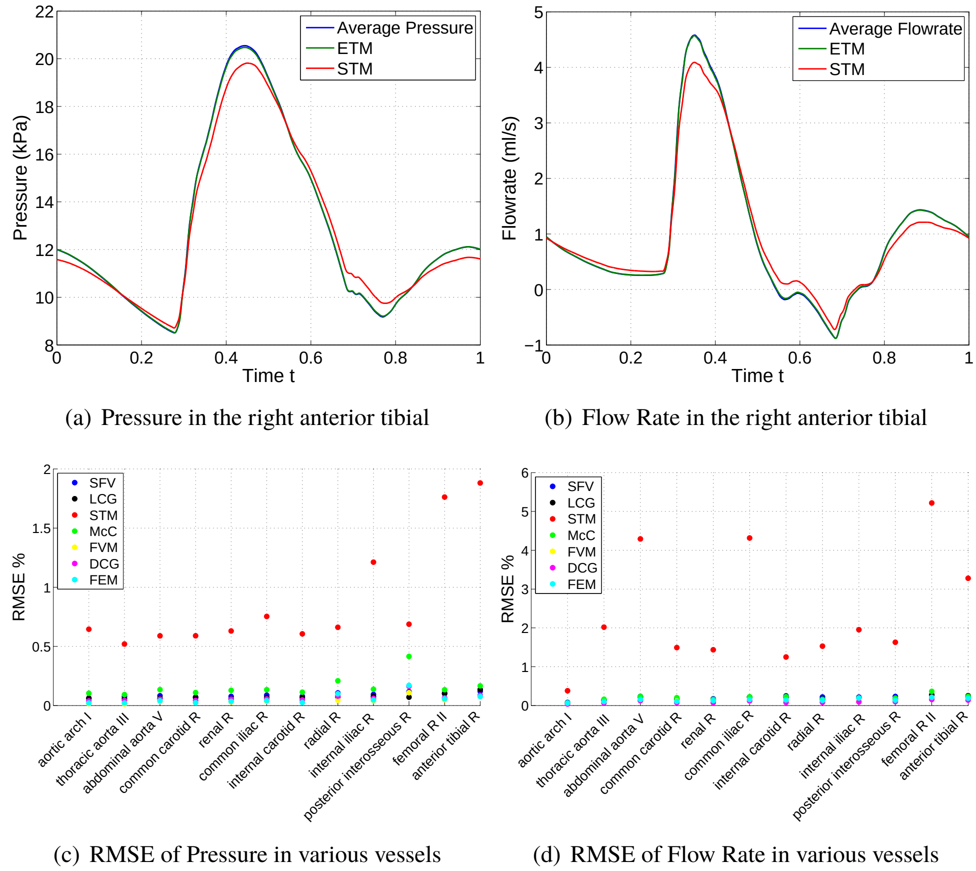
The heart and valve models are similar to those used in [5], which had been originally developed in [4]. The heart chambers are described using elastance curves $E_{ch}$ built from two Hill functions. These elastance curves are used to construct the native $E_{nat}$ and septal $E_{sept}$ elastances. The pressure in a heart chamber is determined from

$$P_{ch} = E_{nat}\left(V_{ch} - V_{0,ch}\right)\left[1 - K_{s,ch}Q_{ch}\right] + \frac{E_{nat}}{E_{sept}}P^*, \tag{4}$$

where $V_{ch}$ is the volume of the chamber; $V_{0,ch}$ is the residual volume; $K_{s,ch}$ is a constant; $Q_{ch}$ and $P_{ch}$ is the flow rate and pressure in the heart chamber respectively; and $P^*$ is the pressure in the contralateral chamber.

The valve model uses an opening coefficient $0 < \zeta < 1$ where

$$\frac{\partial \zeta}{\partial t} = K_o(1-\zeta)\Delta P, \qquad \frac{\partial \zeta}{\partial t} = K_c(\zeta)\Delta P. \tag{5}$$




(a) Pressure in the right anterior tibial

(b) Flow Rate in the right anterior tibial

(c) RMSE of Pressure in various vessels

(d) RMSE of Flow Rate in various vessels

Figure 2: Pressure and flow rate waveforms and error comparison between ETM, STM and Averaged solution of methods from [1]

The relationship of this coefficient for varied pressure differences across the valves is shown in figure 1(b). This coefficient is used to determine an effective area, which, in turn is used to calculate the Bernoulli resistance $B$ and inductance $L$. These are used to updated the flow rates using

$$\Delta P = BQ\,|Q| + L\frac{\partial Q}{\partial t}. \tag{6}$$

The implicit solver used is the enhanced trapezoidal method (ETM), which is a modified version of the simplified trapezoidal rule method (STM) [2]. The method relies on the ability to write the governing elemental equations in the form

$$\boldsymbol{F}_e \boldsymbol{P}_e^{n+1} + \boldsymbol{G}_e^c \boldsymbol{Q}_e^{c,n+1} = \boldsymbol{h}_e^n. \tag{7}$$

The method uses second order backward differences in time and the trapezoidal rule in space.

At vessel junctions the conservation of mass (8) is satisfied via the formulation, and, hence only conservation of total pressures (9) are held as system constraints. The constraints are held in place using using Lagrange multipliers. For example from a parent vessel ($p$) to $N$ daughter vessels ($d_i$), constraint (9) will need to be satisfied, and hence $N$ Lagrange multipliers will be needed.

$$g_1 = A_p u_p - \sum_{i=1}^{N} A_d^{(i)} u_d^{(i)} = Q_p - Q_{d1} - Q_{d2} = 0, \tag{8}$$

$$g_{i+1} = \frac{\rho}{2}\frac{Q_p^2}{A_p^2} + P_p - \frac{\rho}{2}\frac{Q_{di}^2}{A_{di}^2} - P_{di} = 0, \qquad i = 1, 2, \ldots, N. \tag{9}$$

The ETM scheme has been compared with benchmark problems from [1], and results are in agreement with other commonly used numerical methods.

## 3. Results

The enhanced trapezoidal rule method (ETM) has be applied to a cardiovascular network containing 56 one-dimensional vessels terminating in a three element Windkessel model, given in the benchmark paper [1]. For a time step of $\Delta t = 0.5$ ms and maximum element size of $\Delta x = 0.5$ mm the method gives expected results. Figure 2(a) shows the pressure in the right anterior tibial artery for the last cardiac cycle. The flow rate in the right anterior tibial artery is shown in 2(b). The root-mean-square error (RMSE) of pressure and flow rate waveforms for vessels which are monitored are shown in figures 2(c) and 2(d) respectively. The scheme was implemented in MATLAB Student R2013a (The MathWorks, Inc., Natick, MA, USA) on an Intel Core i5-3337U 1.8GHz. The code ran for 20 seconds per cardiac cycle and needed 8 cardiac cycles to converge to the desired tolerance.

## 4. Conclusions and Future Work

A 1D closed loop model using the ETM scheme is presented as an improvement to the STM [2]. The scheme has been compared with benchmark problems and gives excellent agreement with other commonly used schemes. Root mean square errors (RMSE) of the STM method vary up to 6.3% for flow rates and 2.3% for pressures. The new ETM method shows errors (whilst using the same time step and element size as the STM), that are less than 0.5% for flow rates and 0.25% for pressures.

The ETM scheme is being applied to a modified version of the model by [4], which is a closed loop system consisting of 396 1D vessels. The closed loop model is to be the basis for future research which involves investigating changes in the cardiovascular system during pregnancy. This will include the addition of ovarian and uterine arteries/veins, which increase significantly in diameter. The effects of a growing foetus will also investigated. Experimental data will be provided in order to aid the development and fine tuning of the model.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