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MODELLING SCHEME FOR RAILWAY VEHICLE/TRACK/GROUND DYNAMIC INTERACTION IN THE TIME DOMAIN

J. Y. Shih[1*], D.J. Thompson[1], A. Zervos[2]

[1]Institute of Sound and Vibration Research, Faculty of Engineering and the Environment, University of Southampton, Southampton, SO17 1BJ, UK.
[2]Infrastructure Research Group, Faculty of Engineering and the Environment, University of Southampton, Southampton, SO17 1BJ, UK.

*js10e12@soton.ac.uk

ABSTRACT

Modelling of vehicle/track/ground dynamic interaction is an important issue for railway design. Better understanding of how the moving dynamic loads are distributed through the track components to the ground can be derived from these numerical results to improve the stability of the moving train and decrease the cost of the maintenance. Nonlinear models of the ground may be required due to the large displacements induced by heavier and/or high-speed trains. The aim of this research is to develop a general modelling approach for predicting the dynamic behaviour for a variety of situations.
A three-dimensional vehicle/track/ground approach in the time domain is presented. The finite element method is used to model the track/ground vibration. The equations of motion of a multi-body vehicle are implemented to couple with the ground/track system. An alternative approach to the commonly used infinite elements is proposed for modelling the far-field, based on the use of mass-proportional damping to suppress the reflections from model edges. Improved results are shown and better efficiency can be found compared to the results from models with infinite elements. Furthermore, two different geometries for the ground model, a hemispherical and a cuboid one, are discussed. The issue of transients developed by the moving load is discussed and it is shown that long models are required for load speeds close to the wavespeed in the ground to allow the results to achieve steady state. Finally, the results are benchmarked against the results from a wavenumber FE/BE model.

Keywords: *Vehicle/track interaction; Finite Element (FE); viscous damping; ground vibration; track dynamics*

1. Introduction

To achieve better understanding of the ground-borne vibration induced by high-speed trains, numerical simulation has become very important, allowing investigations of the dynamic behaviour of the track/ground system when the train is passing. Assessment of critical speed, ballast and soil degradation, and environmental impact all rely on results from such simulations. To prevent spurious reflections from the domain boundaries, a semi-infinite domain model is required for the soil. Several numerical methods have been developed recently that incorporate absorption at the boundary. Boundary elements (BE) directly include the infinite medium and these can be combined with finite elements (FE) either in a fully three-dimensional FE/BE method [1], or in a so-called 2.5D FE/BE method [2]-[5]. However, these methods mostly operate in the frequency domain and consequently cannot account for the nonlinear behaviour that may occur due to heavy axle loads or higher train speeds. Therefore, an alternative scheme has become common for modelling ground vibration induced by moving loads in which the FE domain is bounded by infinite elements [6]–[9]. Infinite elements form a typical local boundary method that uses viscous dashpots to absorb the incident waves at the boundary. They are defined based on the following equation

$$\begin{cases} c_{Ni} = \rho c_p \\ c_{Ti} = \rho c_s \end{cases} \tag{1}$$

where c_{Ni} and c_{Ti} are the damping values for the normal and tangential direction, respectively. ρ is the material density and c_p, c_s are the wave speeds of the P-wave (longitudinal wave) and the S-wave (shear wave). However, infinite element method does not generally have perfect absorption at the boundary. The absorption efficiency of the infinite elements relies on the domain geometry and the incident wave field. Usually they should be located in the far field and orientated perpendicular to the incident waves. The aim of the present study is to investigate the limitations of the use of infinite elements and to determine the optimum modelling approach for track/ground vibration induced by moving dynamic loads. Comparisons are made with a 2.5D FE/BE approach [5].

2. Numerical models

The purpose of the study is to investigate critical velocity effects, when the load speed approaches the speed of waves in the ground. First, however, the model is considered for a stationary harmonic load.

Two different geometries for the ground model are used for modelling the soil: a hemispherical one (Fig. 1(a)) and a cuboid one (Fig. 1(b)). Infinite elements are applied around the boundary of the hemispherical model. Even though a hemispherical model can show better absorption efficiency of outgoing waves, incorrect artificial displacements of the whole model occur due to the fact that the infinite elements are statically unconstrained [10]. The cuboid model therefore has a fixed bottom to avoid this incorrect displacement. Fixed boundaries are also used at the ends of the model while along the sides infinite elements are considered as an alternative to fixed boundaries. In all cases symmetry of the model is used along the track centreline. Furthermore, when the load speed approaches the critical speed, a relatively long model is required to allow the results to achieve steady state, especially for homogeneous soil models [10]. As a result, the cuboid model is preferred for the moving load problem as the required geometry can be generated relatively easily.

A viscous damping model, based on Rayleigh damping, is used here for the calculations with the FE model. The Rayleigh damping is based on two parameters α and β, which allow the damping matrix **C** to be determined from the mass and stiffness matrices, **M** and **K**:

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{2}$$

This allows the equivalent loss factor η or damping ratio ξ to be obtained as a function of frequency:

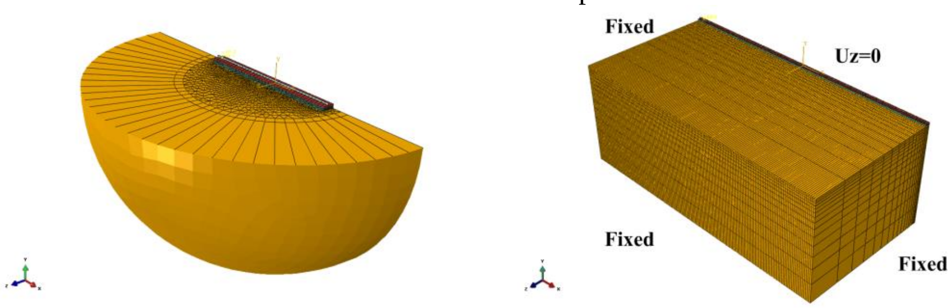
$$\frac{\eta}{2} = \xi = \frac{\alpha}{2\omega} + \frac{\beta\omega}{2} \tag{3}$$

where ω is the circular frequency at which the loss factor η applies. For a plane harmonic wave at circular frequency ω propagating in an elastic medium at a constant wave speed c, the wavenumber k is given by $k = \omega/c$. In the presence of damping with a damping ratio ξ the wavenumber becomes complex with the form $\bar{k} = k(1-i\xi)$. The imaginary part is related to the decay with distance which can be expressed in dB/m as

$$D = -20\log_{10}\left(\left|e^{i\bar{k}}\right|\right) = -8.69\,\mathrm{Im}(\bar{k}) = 8.69k\xi \tag{4}$$

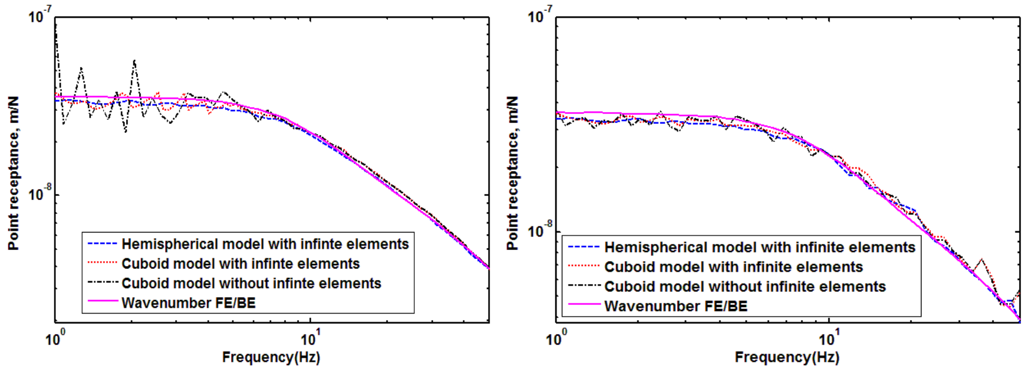
From Eq. (3), the decay with distance for stiffness-proportional damping is proportional to the square of the frequency, $D = 4.34\omega^2\beta / c$. So, for example, for β = 0.000636 (equivalent to η = 0.05 at 12.5 Hz) the decay with distance is around 0.002 dB/m at 1 Hz rising to 5 dB/m at 50 Hz. For the case of a constant loss factor $\eta = 2\xi$, the decay with distance is found to be $D = 4.34\omega\eta / c$ which increases in proportion to the frequency. For mass-proportional damping D is independent of frequency, $D = 4.34\alpha / c$. Thus, e.g. for α = 0.98, D is 0.075 dB/m for all frequencies. As a result it can be expected that mass-proportional damping will be equally effective at suppressing reflections from the domain boundaries at all frequencies.

Fig. 2 shows the point receptance of the rail due to a harmonic load for the various different ground models shown above. The soil is a homogeneous half-space with shear wave speed 60 m/s. The hemispherical domain has a radius 40 m; the cuboid models have a length of 80 m and width of 40 m for the case without the infinite elements and 20 m width for the one with infinite elements. In each case the force is applied at the centre of the rail. Two different viscous damping models are used: stiffness-proportional damping with α=0 and β=0.000636 and mass-proportional damping with α=0.98 and β=0. The results from a 2.5D FE/BE model are shown for comparison, in this case with constant loss factor η = 0.05. This shows that the hemispherical model with infinite elements gives much better results than the cuboid model particularly at low frequency. Furthermore, the results with mass-proportional damping agree better than the results with stiffness-proportional damping. Even though the results from the cuboid model with infinite elements show less fluctuation than those without the infinite elements, some small oscillations still can be found. The cuboid model without infinite elements is preferred as, although the model with infinite elements gives improved results, it is much less efficient for the moving load problem. For example, for a load speed of 57m/s, the 40 m wide cuboid model requires 7.8 hr calculation time compared with 19 hr for the 20 m wide model with infinite elements and 12.5 hr for the 40 m radius hemispherical model.



(a) (b)

Figure 1. Ground model; (a) hemispherical ground model; (b) cuboid ground model



(a) (b)

Figure 2. Point receptance on the rail for homogeneous half-space; (a) stiffness-proportional damping; (b) mass-proportional damping

Finally, results are shown from a time-domain analysis for various speeds. Fig. 3 shows results for the homogeneous half-space (shear wave speed 60 m/s) whereas Fig. 4 shows results for a layered half-space with a 2 m deep upper layer of the same soft soil overlying a stiffer soil with a shear wave speed of 120 m/s. Good agreement can be seen with the results from the wavenumber FE/BE model [5], except close to the critical speed 57 m/s (equal to the Rayleigh wave speed in the ground) as shown in Fig. 3a. On the other hand, for the results from the layered half-space good agreement is seen even close to the critical speed (which here is around 85 m/s).

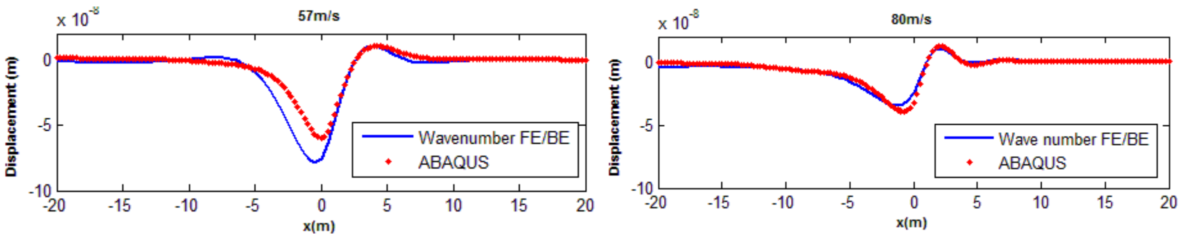


Figure 3. Comparison between the results from FE model and the results from wavenumber FE/BE for homogeneous half-space; (a) V=57 m/s; (b) V=80 m/s

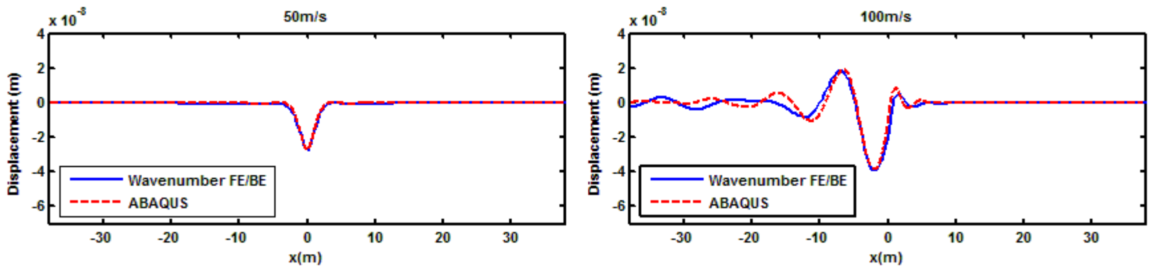


Figure 4. Comparison between the results from FE model and the results from wavenumber FE/BE for layered half-space; (a) V=50 m/s; (b) V=100 m/s

3. Conclusion

An investigation has been presented of the use of the time-domain finite element approach to represent a load moving along a railway track on a flexible ground. The results from a hemispherical model with infinite elements show better absorption efficiency than a cuboid model. However, incorrect displacements are found due to the infinite elements. A cuboid model with fixed boundaries with mass-proportional damping is recommended for modelling the moving load problem due to its relatively accuracy and efficiency. Although the results shown are for linear soil models, the time-domain approach can be readily extended to consider nonlinear soil models.

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