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Fracture models for hydraulic fracturing stimulation: Comparison between Numerical method and an EPR-based method

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ABSTRACT

In this paper, the behaviour of fractures and their effect on the stress state in a reservoir is investigated as one of the key factors to predict fractures evolution throughout the process. For the numerical modelling, an Extended Finite Element (XFEM Model) has been developed. Also, a model based on an EPR-based (data mining) method has been adopted to predict the fracture behaviour. The data collected from numerical simulations are used to train the EPR-based Models [1].

Keywords: *Fracture model, XFEM, EPR*

1. Introduction

Stress state in geological formations going through hydraulic fracturing is significantly different from stress state in a normal geological formation especially in the zones near a wellbore. The drilling of a well or injecting high pressured fluid into a formation can form a complex stress regime close to a wellbore [6]. In order to study this near wellbore stress state, it is necessary to investigate the evolution of hydraulic fractures in a formation.

This research attempts to make a comparison between a numerical model and a data mining model to capture the fracture behaviour. First, an Extended Finite Element (XFEM) model has been developed to simulate the fracture response and calculate the stress fields and displacement in a fractured medium. The XFEM model is an elastic 2D model under plain strain and quasi-static conditions. Fracturing process is modelled based on linear elastic fracture mechanics (LEFM) theory. XFEM allows discontinuities to be modelled within the finite element mesh and fracture geometry can evolve in any step of the simulation without any need of re-meshing of the finite element mesh [3,4]. Also, a new method is adopted to model the fracture response by applying evolutionary polynomial regression technique. The results obtained by the proposed EPR-based model illustrate very good agreement with the numerical results and available analytical solutions. It is possible to generate more than one model for a specific case using EPR approach. These models could have different accuracy their predictions. More detailed discussion regarding the method can be found in [1].

2. Extended Finite Element Model

Different numerical methods have been proposed by researchers to simulate the crack growth problem such as discrete element methods and boundary element methods. In the past, several strategies have been developed within the finite element framework in order to address the feasibility of numerical simulation. Most effective one is the extended finite element method (XFEM). In this approach, discontinuities such as cracks are permitted to propagate independently of the finite element mesh by allowing the crack to cross the elements. In XFEM, the finite element mesh will be enriched by additional degrees of freedom on the nodes affected by the cracks (figure 1).

XFEM will allow you to preserve the symmetry and sparsity of stiffness matrix, arbitrary crack geometry to the mesh and employing automatic enforcement of continuity [3,4]. The extended finite element (XFEM) approximation to calculate the displacement of a certain point (x) in a domain with a discontinuity is [2]

$$u(x) = u^{FE} + u^{enr} \tag{1}$$

$$u(x) = \sum_{I\in N} N_I(x)u_I + \sum_{I\in N^{cr}} \tilde{N}_I(x)(H(x) - H(x_I))a_I + \sum_{I\in N^{tip}} \tilde{N}_I(x) \sum_{k=1}^{4} (F^k(r,\theta) - F^k(x_I))b_I^k \tag{2}$$

Where x is the position vector; N_I and $\tilde{N}_I$ are the standard and enriched shape functions; N is the number of nodes in the finite element domain; N_{tip} is the number of nodes of elements containing a crack tip; N_{cr} is the number of nodes of elements cut by a crack but do not contain a crack tip.

In XFEM, The nodes of an element cut by a discontinuity are enriched by Heaviside function [2]

$$H(x) = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases} \tag{3}$$

And the nodes belong to an element containing a crack tip are enriched by the crack tip enrichment functions defined as [2]

$$F^1(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}, F^2(r,\theta) = \sqrt{r}\cos\frac{\theta}{2}, F^3(r,\theta) = \sqrt{r}\sin\frac{\theta}{2}\sin\theta, F^4(r,\theta) = \sqrt{r}\cos\frac{\theta}{2}\sin\theta \tag{4}$$

Where r and θ are local polar coordinate system at the crack tip.

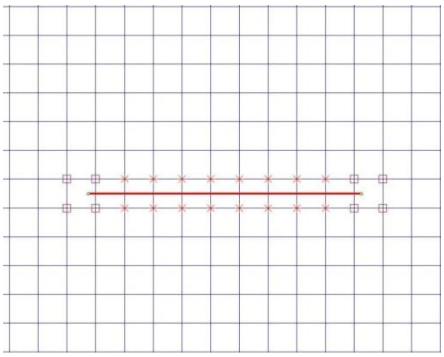

Figure 1. Enriched finite element mesh. Nodes enriched by tip functions are marked with red squares. Nodes enriched by step function are marked with red crosses.

3. Evolutionary Polynomial Regression Model

Evolutionary polynomial regression (EPR) is a data mining method based on evolutionary algorithms. EPR is designed to search for polynomial structures representing a system. A general EPR formulation can be presented as [1]:

$$y = \sum_{j=1}^{n} F(X, f(X), a_j) + a_0 \tag{5}$$

Where y is the estimated vector of output of the process; a_j is a constant; F is a function constructed by the process; X is the matrix of input variables; f is a function defined by the user; and n is the number of terms of the target expression. In EPR algorithm, the evolutionary starts with computing a constant mean of output values. The number of parameters in the model equations will increase by the raise in the number of evolutions in the process. The accuracy of each EPR model can be determined based on coefficient of determination (COD) as the fitness function [1]:

$$\mathrm{COD} = 1 - \frac{\sum_{\mathrm{N}}(\mathrm{Y_a} - \mathrm{Y_p})^2}{\sum_{\mathrm{N}}(\mathrm{Y_a} - \frac{1}{\mathrm{N}}\sum_{\mathrm{N}} \mathrm{Y_a})^2} \tag{6}$$

Where Y_a is the real output value; Y_p is the output predicted by EPR model and N is the number of data points used to determine the COD. Any EPR-model which does not reach an adequate value of fitness or other termination conditions, maximum number of generation and maximum number of terms, needs to go through another evolution loop to obtain a new model [5].

4. Results

A number of conventional fracture mechanics problems are simulated by XFEM. Figure 2 shows a tensile plate with an edge crack. The analytical solution for the mode I stress intensity factor for this example is

$$K_I = [1.12 - 0.23(c) + 10.56(c)^2 - 21.74(c)^3 + 30.42(c)^4]P\sqrt{\pi a} \tag{7}$$

Where P is the applied tensile stress; $2a$ is the crack length and c is the ratio between the crack length and width of the plate. The following range of inputs are used in the XFEM simulation, $250 \leq P \leq 2500\ psi$ with increment of $\Delta P = 10\ psi$ which will provide 226 unique values for the stress value. Also, $0.05 \leq c \leq 0.45$ and increment of $\Delta c = 0.025$ gives 17 different values. The EPR-based models are computed based on the results determined by XFEM. The size of the dataset used in the EPR-based analysis is 3842. About 85% of the data is used to train the EPR model and 15% for validation of the models.

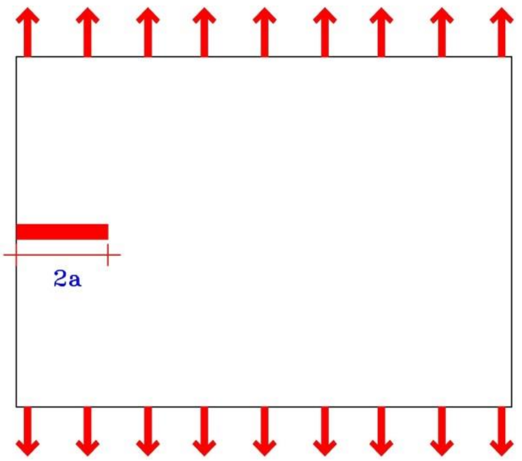

Figure 2. Tensile Plate with an edge crack.

Table 1 Shows 6 models produced by EPR. The EPR model with higher COD and less complexity is the best choice [5]. Stress intensity factor (K) values predicted by best EPR model show a good agreement with the numerical results and exact analytical solutions (see Figure 3).

Conclusion

Fracture models have been developed using evolutionary polynomial regression method. Models predict stress intensity factor values at crack tips. The results demonstrate that EPR models can predict stress intensity factor values effectively and accurately. EPR-based fracture models can be implemented into numerical calculations to decrease the computational time and effort.

Table 1: EPR expressions

EPR-based equation	COD
$K = 114.72P^{0.5}$	60.91
$K = 5.598Pc^2 + 179.9$	96.31
$K = 234.06P^{0.5}c^{0.5}$	80.64
$K = 1.9758P + 11.067Pc^2$	99.96
$K = 400.53c^3 + 1.9776P + 10.96Pc^2$	99.96
$K = 1.8862P + 2.06Pc + 16.08Pc^3$	99.96

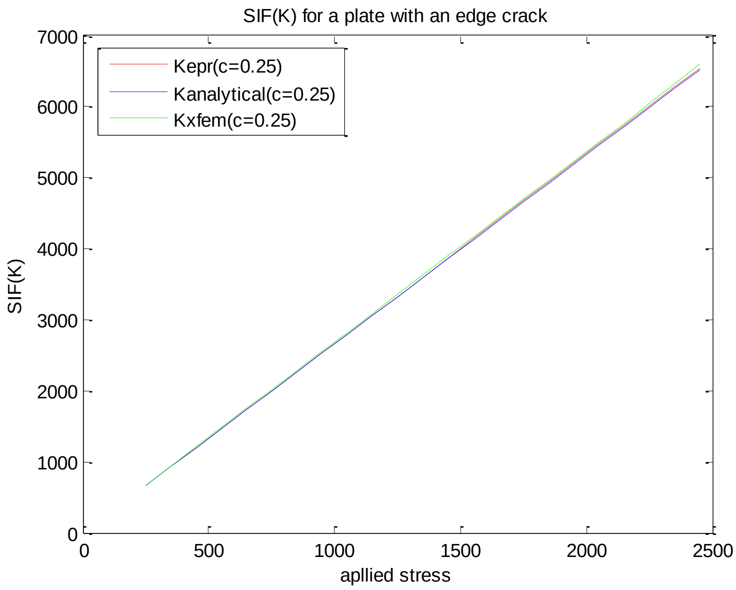

Figure 3. Stress intensity factor (K) vs applied stress

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