

# **MODELING INERTIAL FORCES ON CYLINDERS IN CROSS FLOW USING MOVING FRAME OF REFERENCE**

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## **Abstract**

Fluid-Structure interaction (FSI) problems have received significant attention in many applications in the recent decade. An FSI problem can be simulated either in an inertial frame of reference or in a moving (non-inertial) frame of reference. In the latter case, which is mainly applicable when there is only one moving body, the frame of reference in which the governing equations are solved is attached to the moving body.

In this paper, an immersed boundary (IB) interpolation approach is used to simulate fluid flow. A comprehensive parametric study is performed to find the optimum mesh size and computational domain extent. In addition, the effect of the fluid inertial force in a moving frame of reference is investigated by studying hydrodynamic forces. It is shown that the difference between the governing equations in the relative and inertial frames of reference are the fluid inertial forces which can be added separately to the simulation results. In addition, results of simulations using two frames of reference are contrasted. It is shown that the main sources of noise in the lift coefficient are the pressure fluctuations in the non-inertial frame of reference.

**Key words:** Immersed-Boundary, interpolation/Reconstruction method, pressure gradient, Vortex shedding, FSI

## **1. Introduction**

Fluid flow around a moving bluff body with an irregular geometry has been extensively studied and can be categorised in various ways [<sup>1,2</sup>]. Regardless of the simulation approach (conforming grid, e.g. ALE or non-conforming grid, e.g. IB), it is possible to simulate moving boundaries in either inertial or non-inertial frames of reference. Here an immersed boundary (IB) method is applied in both frames of references [<sup>3</sup>].

Paskin [<sup>4</sup>] was the first to employ the immersed-boundary method for solving flow problems in regions with complex/moving boundaries. These methods are normally classified, by the way that the solid boundary is enforced, into continuous and discrete forcing approaches. In the later scenario, the interpolation/reconstruction method is a popular approach: When the boundary does not align with a mesh line, the solution algorithm is locally replaced by a velocity interpolation to enforce the boundary conditions on the flow field [<sup>5</sup>]. Note that in order to fulfil the conservation of mass near the immersed boundary, extra measures need to be taken (more detail see [<sup>5</sup>]).

When there is only one moving body, employing a relative frame of reference that is attached to the body could improve non-conforming grid approaches significantly as there is no need to update the IB formulation due to movements of the body relative to the background computational grid.

To solve the governing equation in the relative frame of reference two fundamental changes are necessary. First of all, the governing equation should be adapted for usage in the relative frame of reference [<sup>6</sup>] by adding additional terms to compensate for the effect of the moving frame in the calculation. Also, the boundary conditions should be transformed for usage in the relative reference. Here, only movements in the transverse direction are considered.

## **2. Numerical model**

This section briefly outlines the theory and governing equations necessary to simulate a moving body in inertial and non-inertial frames of reference. For reasons of simplicity and easiness of generalization of the solution algorithm, the non-dimensional form of the Navier-Stokes equation is used. The non-dimensional Navier-Stokes equations in primitive variable are given in equation (1). In the moving frame of reference the acceleration of the structure needs to be added to the governing equations and the velocity boundary conditions will also need to be transformed to the relative frame (for more details see [<sup>6</sup>]).

The flow governing equations in both frames of reference are solved by the fractional step (Chorin projection approach) method. The intermediate velocity is calculated at first instance using a Runge-Kutta method for integrating the momentum equations in time. The velocity is subsequently projected

onto a solenoidal space by adding the pressure obtained by solving the pressure Poisson equation (PPE), which is solved using a SIP solver. The Neumann condition for the PPE should be updated by projecting the momentum equation at the immersed boundary to account for the solid acceleration [6].

$$\begin{aligned} \frac{\partial V}{\partial t} + V \cdot \nabla V &= -\nabla p + 1/Re \nabla^2 V \\ \nabla \cdot V &= 0 \end{aligned} \quad (1)$$

### 3. OSCILLATION OF A BODY IN CROSS FLOW DIRECTION

In this problem, the body oscillates with a prescribed motion in the cross flow direction at specific range of oscillations where the frequency of vortex shedding around the body becomes similar to the oscillation frequency ('lock-in' phenomenon). Here, the prescribed motion is defined in equation (2), where  $y_c(t)$  is the location of the centre of the cylinder and  $A_{0y}$ ,  $\omega$  and  $f$  are the amplitude, the frequency in rad/s and the frequency in Hz of the prescribe oscillation, respectively.

$$y_c(t) = A_{0y} \text{Sin}(\omega t) = A_{0y} \text{Sin}(2\pi f t) \quad (2)$$

In the moving frame of reference approach the reference frame is attached to the moving body and the boundary conditions are transformed accordingly. Note that the flow about a circular cylinder forced to oscillate in the cross flow direction is kinematically the same as the flow about a fixed cylinder in a free stream with a superimposed oscillatory cross flow. It should be noted, however, that these two flows differ dynamically due to the inertial effects. This effect is known as the Froude-Krylov force in the literature [7]. To simulate flow over a moving body using an IB approach in an inertial frame of reference the interpolation formula needs to be regularly updated, while in the moving frame of reference the interpolation formula remains unchanged. The main advantage when using an IB approach is the ability to simulate the Fluid-Structure-Interaction (FSI) for a moving/deforming object on a fixed grid.

### 4. Results

Various parameters, which could potentially affect the hydrodynamic forces exerted by the uniform stream on an oscillating cylinder, are studied. According to the parametric study for flow around a moving cylinder with prescribe motion, the mesh size, the domain size upstream of the cylinder and the domain size in the cross stream direction are the most influential factors. Here, these effects are studied for a cylinder oscillating in the cross flow direction with an amplitude of  $A/D=0.2$ , while the frequency of excitation is  $f_e=1.05f_s$ . The parametric study is performed at  $Re=100$ , based on the free stream velocity and the cylinder diameter, where  $f_s=0.167$  is the Strouhal frequency for a stationary cylinder.

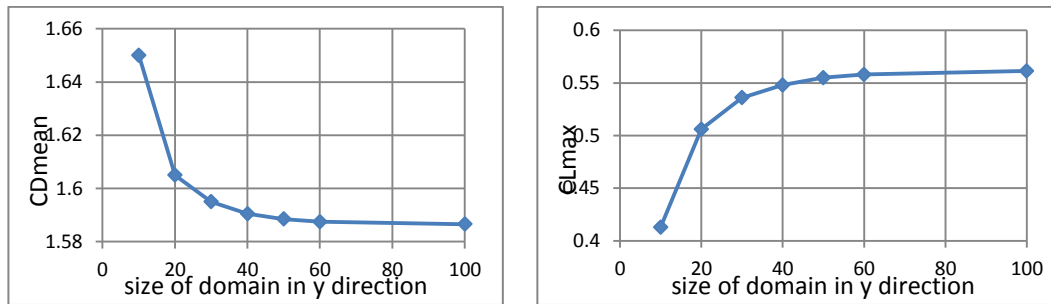


Figure 1: The effect of domain size in the y direction on the mean drag and maximum lift

The numerical results show that lift and drag coefficient changes become negligible (less than 1%) for the grid sizes finer than  $0.025D$ . Also, if the size of the domain upstream of the cylinder is increased from  $20D$  from  $30D$ , the mean drag and maximum lift coefficients only change by  $0.2\%$  and  $-1.5\%$  respectively. In addition, the blockage effect on the lift and drag coefficient is less than  $1\%$  when the size of the domain in the transverse direction exceeds  $40D$  (Figure 1).

#### 4.1 Effect of inertial forces

When the flow governing equation is solved using relative velocities rather than absolute ones, without deriving the equations in a moving frame, inertia effects need to be added to the relative hydrodynamic forces (case A). On the other hand, if the flow governing equations are derived in the moving reference frame the flow forces will include inertia effects (case B). This has been shown Figure 2, where in both simulations,  $Re=150$ ,  $F = f_e/f_s=0.9$  and the relative velocities are defined at the inlet and far-field boundaries (top and bottom).

In both cases (Case A and B) the pressure, the lift due to shear, the drag coefficient due to pressure and the shear stress are found to be identical (**Error! Reference source not found.**). The only difference is in the lift due to the pressure which is caused by the inertial force. In Figure 2 (left), the red line and the green line show the lift coefficient (due to pressure) for cases A and B, respectively. In this figure, once the lift in case A (red line) and the inertial effect (orange line) are added together (back dots), the results become in good agreement with the lift coefficient obtained in the case B (Green line).

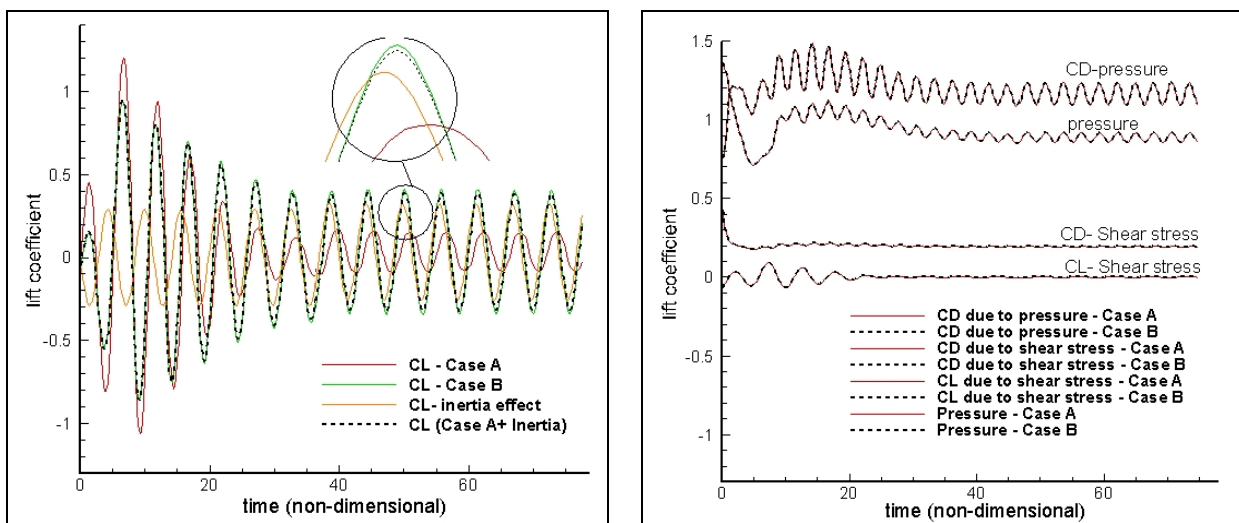


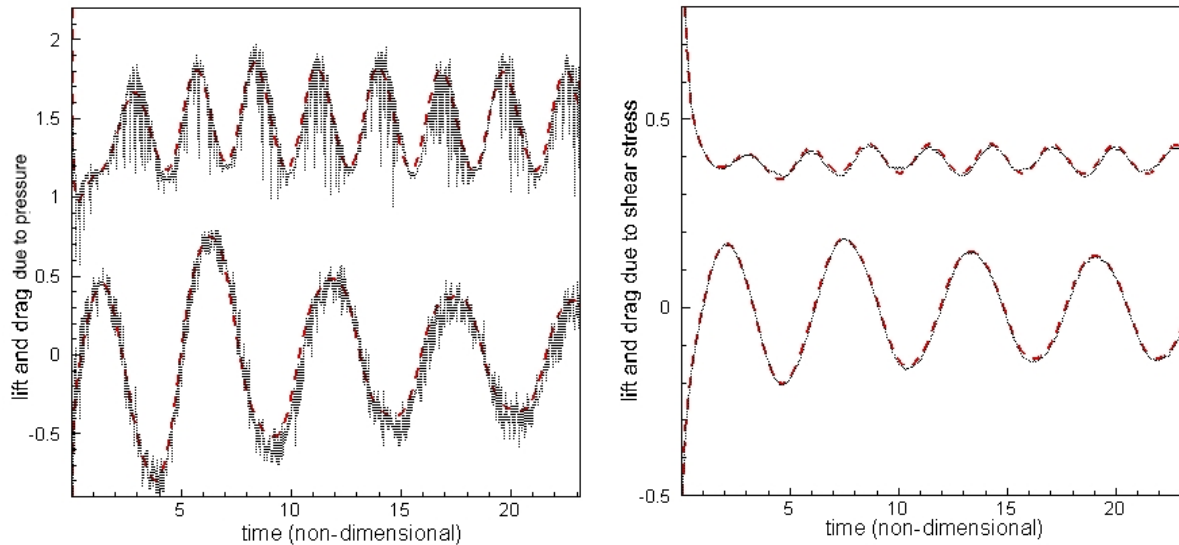
Figure 2: inertial effect to correct lift coefficient calculated in moving frame of reference

#### 4.2 Moving frame versus inertial frame of reference

The governing equations are solved in both the moving frame of reference and the inertial frame of reference. In both approaches an IB interpolation method is used to enforce the immersed boundary. In the moving frame of reference, however, the interpolation formulas are not updated so that the simulation is less time consuming and the results are much smoother. In both cases a cylinder is forced to oscillate in the cross flow direction, the Reynolds number is 100 ( $Re=100$ ), the amplitude of the oscillation is  $0.2D$  and the frequency of the oscillation is 1.05 times the vortex shedding frequency (0.167). The Reynolds number is based on the free stream velocity and the cylinder diameter,  $D$ .

The lift and drag due to pressure for both approaches (moving and fixed frame of reference) are shown in the Figure 3. The results from the inertial frame of reference simulations show noise in the lift and drag signals due to pressure (dotted line). The reason for this is that the interpolation formulas are updated at each time step.

The lift and drag due to shear stress for both non-inertial and inertial frames of reference are similar and do not show any noise (Figure 3, right). It can be concluded that the noise in the lift and drag coefficients are due to the calculation of the pressure. Also this possibly explains why the inertial frame of reference approach is so time consuming. Not only updating the interpolation formulas is taking extra simulation time but also the Pressure Poisson equation needs more iteration to converge due to the noise in the pressure.



**Figure 3: dotted lines, inertia frame of reference; dash lines, moving frame of reference**

## 5. Conclusion

The flow around a circular cylinder with a prescribed motion in cross flow using an IB approach is studied in inertial and non-inertial frames of reference. A comprehensive parametric study is performed to find the optimum mesh size and to define the size of the computational domain. The effect of flow inertial forces is presented in both frames of reference. Also the lift and drag coefficients were compared in detail and source of the noise in the lift and drag coefficients was identified to be due to spurious pressure fluctuations caused by the regular changes of interpolation parameters at the IB.

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