

# MULTI-SCALE FINITE ELEMENT BASED TIME-DEPENDENT RELIABILITY ANALYSIS FOR LAMINATED FIBRE REINFORCED COMPOSITES

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## ABSTRACT

The objective of this paper is to propose a time-dependent reliability analysis method to investigate the durability of fibre reinforced polymer composites. A stochastic multi-scale finite element method, which is based on computational homogenization and perturbation technique, is adopted to propagate uncertainties in both micro- and macro-scale parameters. The influence of water absorption and heat conduction and the induced degradation of mechanical properties, which is estimated through a hygro-thermo-mechanical model that is integrated into the stochastic multi-scale finite method, is then studied in the framework of time-variant reliability analysis. It is shown that the problem reduces to a sequence of time-independent problems that can be solved using the first-order reliability method. A numerical study is carried out to demonstrate the applicability of the proposed method, and the evolution in time of the probability of failure is computed.

*Key Words:* Composite; hygro-thermal effect; computational homogenization; durability; reliability

## 1. Introduction

Composite materials may provide benefits in terms of reduction of construction time and carbon emission for civil engineering sector, given their success in other sectors. However, civil engineering structures and infrastructures are expected to serve in natural environment for 50 years or more. Accurately quantifying long term durability behaviour is eagerly needed to fully exploit benefits that composite materials could bring to construction sector. In an on-going UK EPSRC funded research project - *Providing Confidence in Durable Composites (DURACOMP)*, pultruded glass fibre reinforced composites, which are typical construction composite materials, have been tested to characterize their ageing behaviours in hot/wet condition. The evolution in time of moisture uptake ratio and mechanical properties has been measured and reported in [1, 2]. To numerically simulate these coupon testing based findings to large scale structures, a coupled hygro-thermo-mechanical model has been developed [3] to investigate the long term durability of composites in a deterministic context. Arising from various sources such as manufacturing process, assembly and quality control limits, composite materials exhibit uncertainties in their material properties. Taking these uncertainties into account in designing composites, such as through the use of reliability based structural design, is essential to ensure that the structures perform with sufficient safety during their service and fully realize the advantages offered by composites. In the present study, this coupled hygro-thermo-mechanical model is combined with a probabilistic homogenization approach developed in [4] to propagate micro-scale uncertainties to the macro-scale in the mechanical problem. This is subsequently integrated with reliability analysis methods to evaluate the reliability of composite structures.

## 2. Time-dependent structural reliability analysis

For a stochastic structure system, when stochastic processes or functions of time are explicitly present in the limit state function (LSF),  $g(\mathbf{x})$ , time-variant analysis is required to quantify the probability of failure of the structure. Accordingly, the  $n$ -dimensional vector  $\mathbf{x}$  that collects all random parameters in

the system is split into  $\mathbf{x}_1$  which is a vector of random variables and  $\mathbf{x}_2(t)$  which is a vector of random processes. The probability of failure,  $p_f$ , of the structure within a time interval  $[t_b, t_e]$  is defined as

$$p_f(t_b, t_e) = Pr(\exists t \in [t_b, t_e], g(\mathbf{x}_1, \mathbf{x}_2(t)) \leq 0) \quad (1)$$

In this study, the time effect is present in the problem due to the degradation of material properties, the LSF  $g$  thus depends on random variables and functions of time. The computation of  $p_f$  as defined in Eq.(1) reduces to

$$p_{f,i} = Pr(g(\mathbf{x}_1, \mathbf{x}_2(t)) \leq 0) \quad (2)$$

which is called instantaneous probability. In a degradation problem, the LSF is monotonously decreasing in time whatever the realizations of the random variables. For each realization of the random vector, the minimum value of  $g(\mathbf{x}_1, \mathbf{x}_2(t))$  over a time interval  $[t_b, t_e]$  is attained for  $t = t_e$ . Thus, the time-variant problem is reduced to a time-invariant analysis to calculate the special case of Eq. (2) at  $t_e$ . First-order reliability method (FORM) is one of the widely used methods to numerically estimate reliability for time independent problem. Essentially, it requires to transform random variables  $\mathbf{x}$  from general probability space to standard normal space as  $\mathbf{u}$ , which leads to  $G(\mathbf{u})$ , and to linearly approximate the LSF through the first-order Taylor series expansion, which needs to calculate gradient of  $G$ . According to the chain rule of differentiation, the gradient vector of  $G$  can be obtained from

$$\nabla G = \frac{\partial G}{\partial \mathbf{y}} = \frac{\partial g}{\partial s} \frac{\partial s}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{y}}, \quad (3)$$

where  $\frac{\partial \mathbf{x}}{\partial \mathbf{y}}$  is Jacobian of the probability transformation,  $\frac{\partial g}{\partial s}$  can be analytically obtained since the LSF  $g$  is function of components of  $s$ . Thus, the only unknown is  $\frac{\partial s}{\partial \mathbf{x}}$  that will be calculated from stochastic finite element method.

### 3. Stochastic multi-scale finite element analysis

#### 3.1. Degradation model for FRP composites in hot/wet condition

Based on experimental data reported in [1], Zahur et al. [3] proposed a generalized model to characterize the degradation of constitutive matrix of matrix material under hygrothermal condition, which is expressed as

$$\frac{d\mathbb{C}_m}{dt} = -c\eta \log(1 - T/T_g) \mathbb{C}_m^0 \quad \text{or} \quad \frac{d}{dt}(1 - \omega) = -c\eta \log(1 - T/T_g)(1 - \omega) \quad (4)$$

where  $\mathbb{C}_m^0$  is for undegraded material;  $c$  is the moisture concentration;  $\eta$  is a model parameter that is obtained from experiment data;  $T$  is the temperature;  $T_g$  is the glassy transition temperature; and  $t$  is the exposure time. Thus, this degradation model is actually to get instantaneous temperature  $T(t)$  and moisture concentration  $c(t)$ , and they are obtained from transient moisture diffusion and heat conduction analyses.

#### 3.2. Computational homogenization for multi-physics problems

Owing to the multi-scale architecture of composite materials, homogenization method should be used to estimate the effective properties of composites. In the present paper, the multi-scale computational homogenization method was used to estimate the effective elastic and physical properties. Thermal and diffusion homogenization were conducted to get the effective conductivity and diffusivity, respectively. These were then supplied to transient heat conduction and moisture diffusion analyses to calculate the instantaneous temperature and moisture concentration for Eq. (4). Finally, the degraded elastic properties of constituent materials were input to the mechanical homogenization to get the effective elastic properties. In the finite element context, the mechanical homogenization is expressed as

$$\begin{bmatrix} \mathbf{K}_\mu & \mathbf{P}^T \\ \mathbf{P} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{D}\bar{\boldsymbol{\varepsilon}} \end{Bmatrix} \quad \text{or} \quad [\hat{\mathbf{K}}] \{\hat{\mathbf{u}}\} = \{\hat{\mathbf{F}}\}. \quad (5)$$

where  $\lambda$  is the Lagrangian vector,  $\bar{\boldsymbol{\varepsilon}}$  is the prescribed or given strain, and  $\mathbf{P}$  and  $\mathbf{D}$  are constraint matrix and global coordinate matrix.

### 3.3. Uncertainty propagation with the use of perturbation technique

Considering material properties as random variables, the stiffness matrix  $[\hat{\mathbf{K}}]$  and the nodal displacement vector  $\{\hat{\mathbf{u}}\}$  in Eq. (5) are thus stochastic functions. According to the perturbation technique, stochastic function can be approximated by Taylor series expansion. In the present study, only the first order expansion is needed as the first order derivatives of structural responses are required in Eq. (3). By expanding stochastic functions in the form of

$$\phi(\mathbf{b}) = \phi(\bar{\mathbf{b}}) + \epsilon \sum_{i=1}^n [D_{b_i} \phi(\bar{\mathbf{b}})] \delta b_i, \quad (6)$$

substituting them into Eq.(5) and equating terms of equal orders of  $\epsilon$ , we arrive at the following zeroth- and first-order equations:

$$[\hat{\mathbf{K}}] \{\hat{\mathbf{u}}\} = \{\hat{\mathbf{F}}\} \quad (7)$$

$$\sum_{p=1}^n \{[\hat{\mathbf{K}}] \{D_{b_p} \hat{\mathbf{u}}\} + [D_{b_p} \hat{\mathbf{K}}] \{\hat{\mathbf{u}}\}\} = \mathbf{0} \quad (8)$$

In Eq. (5), the block related to stiffness matrix,  $[\mathbf{K}_\mu]$ , of the microstructure is function of material properties. It and its first-order partial derivative can be expressed as

$$\mathbf{K}_\mu = \int_{\Omega_\mu^s} \mathbf{B}^T \mathbb{C}_\mu \mathbf{B} dV, \quad \text{and} \quad [D_{b_p} \mathbf{K}_\mu] = \int_{\Omega_\mu^s} \mathbf{B}^T [D_{b_p} \mathbb{C}_\mu] \mathbf{B} dV, \quad (9)$$

where  $\mathbf{B}$  is the strain-displacement matrix,  $\mathbb{C}_\mu$  is the constitutive matrix of constituent material, and  $[D_{b_p} \mathbb{C}_\mu]$  is its first-order partial derivative. Computing Eqs.(7) and (8) successively, the compact displacement vector  $\{\hat{\mathbf{u}}\}$  and its first order partial derivative  $\{D_{b_p} \hat{\mathbf{u}}\}$  can be obtained. These were then used to calculate the effective constitutive matrix and its derivatives according to  $\bar{\mathbb{C}} = \frac{\bar{\sigma}}{\bar{\epsilon}}$  (see [4] for details of the derivation). Once the effective constitutive matrix for the composite is obtained, the stochastic structural responses at macro level can be calculated by introducing a perturbation-based stochastic finite element method, and these are input to Eq. (3).

## 4. Numerical example

A fibre reinforced polymer composite plate is considered in this section to illustrate the time-dependent reliability analysis scheme. The geometry of the plate is shown in Fig 1a. For the macro-level thermal problem, a temperature of  $80^\circ\text{C}$  is applied to the top and bottom surfaces, and constant heat flux is applied to the left and right surfaces. Similarly, for the moisture transport problem, a constant concentration of 1, which represents 100% relative humidity, is applied to the top and bottom surfaces, and constant moisture flux is applied to the left and right surfaces. For the macro-mechanical analysis, a uniform pressure of 1000 MPa is applied along the upper and lower surfaces in the vertical direction to simulate uniaxial tension state. Considering the symmetry of the geometry and boundary conditions for heat transfer, moisture transport and mechanical analysis, only 1/8 of the structure, highlighted in Fig. 1a, needs to be modelled. The plate is made of glass/epoxy plain weave textile composite (see microstructure in Fig. 1b) with elastic and physical properties indicated in [3].

The multi-scale analysis for heat conduction and moisture transport problems were conducted first to get the effective heat conductivity and moisture diffusivity required for performing macro level heat conduction and moisture diffusion analyses. The calculations were conducted in a time interval of 10 days, and a period of 500 days was considered. Thus, a total of 51 times of multi-scale multi-physics calculation were conducted. Using these 51 instantaneous temperature and moisture concentration fields, a series of degradation parameter field was computed by Eq. (4). Then the mechanical computational homogenization was applied to estimate the effective mechanical properties. According to preliminary

study at initial stage, applied load, longitudinal ply strength and longitudinal Young's modulus are the most important parameters in reliability analysis of the considered structures. These three parameters were considered as random variables in the time-dependent reliability calculation, and Tsai-Wu failure theory was used to establish the LSF. Finally, the evolution in time of the reliability index is plotted in Fig. 2. It can be seen that the reliability index decreases about 11% after 500 days degradation.

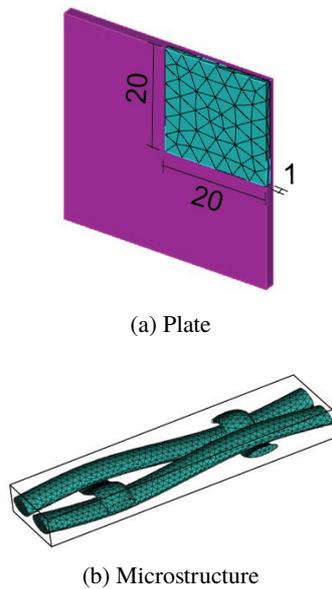


Figure 1: FRP plate and its microstructure

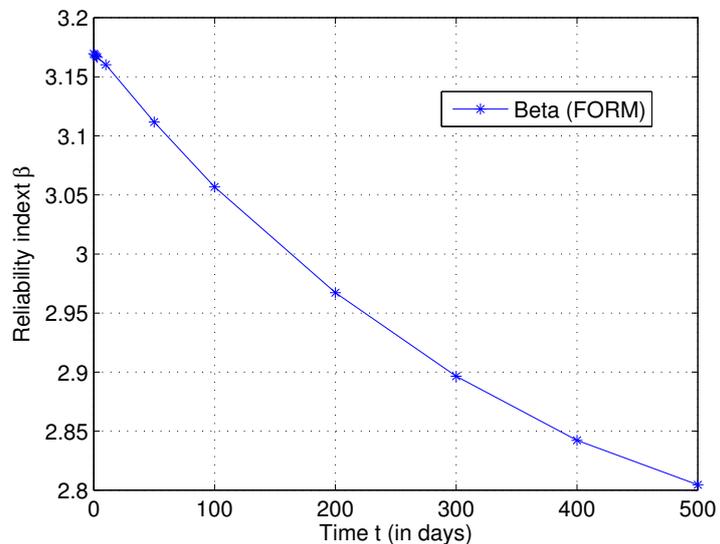


Figure 2: Evolution in time of the reliability index

## 5. Conclusions

On-going research and early results on the durability of fibre reinforced polymer composite structures, which is investigated by time-dependent reliability analysis, are presented in this paper. The time dependent reliability calculation is realized by integrating a multi-scale finite element reliability method with a hygro-thermo-mechanical model. A numerical study is carried out to demonstrate the applicability of the proposed scheme through the calculation of the evolution in time of the probability of failure.

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