

Two level layup optimization of composite plate using lamination parameters

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ABSTRACT

This paper presents a technique for the global layup optimization of laminated composite plates with initial buckling constraints based on lamination parameters. The method is computationally more efficient than previous solutions. The optimization problem is divided into two stages with continuous optimization followed by discrete optimization, in order to achieve the final optimum layup effectively using 0, 90, 45 and -45 plies. During the first stage, the optimal plate thickness and lamination parameters are obtained using VICONOPT, an exact finite strip software for the optimisation of prismatic plates and stiffened panels. In the second stage, a Matlab program based on the branch and bound method incorporating a layerwise technique is used to find the optimal stacking sequences to achieve the required lamination parameters obtained from stage 1. Moreover, the Matlab program is able to ensure the layup to be symmetric, balanced or both.

Keywords: layup optimization, lamination parameter, branch and bound, buckling, composite

1. Introduction

Composite material is increasingly used in aerospace structures because of its improved specific properties and ability to be optimized for particular application. Currently, the industry is substituting laminated composites for more conventional aircraft materials, but their optimization is still a challenging task. Miki and Sugiyama [1] used lamination parameters to optimize orthotropic and symmetric composite plates, significantly reducing the number of design variables but the stacking sequence obtained was unable to satisfy the discrete constraint. Yamazaki [2] first solved this obstacle by using a two-step optimization method in which gradient-based optimization was used in the first step, with genetic algorithms (GAs) then used to implement discrete optimization in the second. Herencia et al [3] developed a finite element method with a closed form (CF) solution and an energy method (Rayleigh-Ritz) used to compute buckling constraints in the first step, and then a GA for the second step. In Liu and Butler's work [4], GAs were also applied during the second stage, with VICONOPT (a buckling program based on the exact strip method and Wittrick-Williams algorithm [5]) used to minimise the panel's weight at the first stage. Kennedy [6] used a branch and bound method which combined with a layerwise technique in the second step was able to match the optimal values of lamination parameters found by VICONOPT in the first step.

2. Methodology

In this paper, an alternative two stage process is implemented. During the first stage, lamination parameters are used as the design variables in VICONOPT. A linear optimizer which assumes local linear approximations to the buckling constraints is used in combination with a penalty function [6] to direct infeasible solutions towards feasible ones without violating any of the constraints. Thus the optimal lamination parameters $\{\xi_j^k\}$ ($j = 1,2,3; k = A,B,D$) and the laminate thickness h for the continuous problem are found.

In the second stage, these lamination parameters are used as targets in a Matlab program which then finds the stacking sequences which give lamination parameters closest to these optimum values. The angles of each of the plies are restricted to four choices (i.e. 0° 90° 45° -45°), resulting in constraints

on the lamination parameters [7] described in Equation (1).

$$2|\xi_1^k| - \xi_2^k - 1 \leq 0, \quad 2|\xi_3^k| + \xi_2^k - 1 \leq 0, \quad k = A, D \quad (1)$$

Therefore the feasible region for $\{\xi_j^k\}$ ($j = 1,2,3; k = A, D$) is triangular as shown in Figure 1.

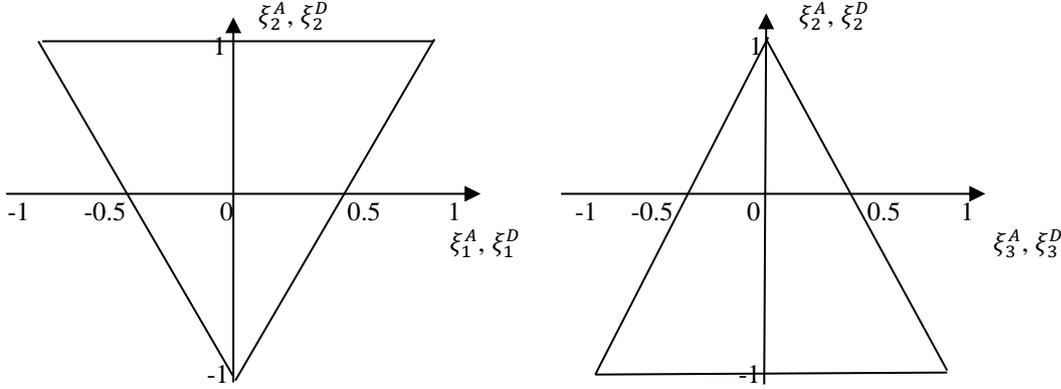


Figure 1: Feasible region for $\{\xi_j^k\}$ ($j = 1,2,3; k = A, D$)

The Matlab program used can be used to constrain the layers to be symmetric, balanced or both. This programme which uses the branch and bound method calculates the contributions to the lamination parameter for each branch in different layers, as obtained from equation (2).

$$\begin{bmatrix} \xi_1^k \\ \xi_2^k \\ \xi_3^k \end{bmatrix} = \int_{-h/2}^{h/2} Z^k \begin{bmatrix} \cos 2\theta \\ \cos 4\theta \\ \sin 2\theta \end{bmatrix} dz \quad (2)$$

where $k = A, B, D, Z^A = \frac{1}{h}, Z^B = \frac{4z}{h^2}, Z^D = \frac{12z^2}{h^3}$, θ is the orientation of the fibres in lamina and z is the depth below the mid-plane.

From the lamination parameters, objective function contributions (Γ) are obtained by calculating the difference between the actual lamination parameter ($\xi_{0^\circ, \pm 45^\circ, 90^\circ}$) and the required lamination parameter ($\xi_{required}$). Then from the summation of the objective function, the route to proceed with for the next branch (i.e. next layer) can be predicted, with the aim being to opt for a branch that is closest to the required lamination parameters.

The layerwise technique employed in this program enhances the optimization process. The ply orientations are successively altered, working inwards from the outer plies which make the most important contributions to the flexural lamination parameters. Initially only two plies are altered at once, then four at once, and so on until in the final cycle all of them can be altered. Thus the branch-and-bound method initially searches the stacking sequences by considering a small number of plies, so that good results can be obtained quickly by solving small problems. Subsequently, when considering larger numbers of plies, the previously found solutions give an upper bound on the objective function enabling many branches to be discarded without being explored. This layerwise approach helps to obtain the optimum layup efficiently whilst achieving a reduction in computational cost.

3. Examples and Results

A simply supported laminated composite plate of width 100mm and length 150mm is subject to longitudinal compression with a compressive load P_d of 10 kN and a shear load P_s of 0.011 kN/mm. The initial buckling load P_c for this plate is 10 kN. The ply thickness h_p is 0.1 mm and the initial laminate thickness is h_i 1.6 mm. The initial ply orientations chosen for this example are arbitrary, the initial lamination parameters corresponding to this initial layup are: $\xi_1^A = 0.0625$, $\xi_2^A = -0.125$, $\xi_3^A = 0.0625$, $\xi_1^D = 0.0127$, $\xi_2^D = -0.0723$, $\xi_3^D = 0.0947$, $\xi_1^B = -0.0078$, $\xi_2^B = 0.1406$, $\xi_3^B = 0.0859$.

For the normal case, the results are illustrated in Table 1. After the first stage, the required lamination parameters are obtained by VICONOPT. Due to the initial buckling load P_c which is equal to the design load P_d , the reduction in laminate thickness is slight, and after rounding the optimized thickness h_0 to the nearest integer number of laminate thickness, the thickness of the optimized plate is 1.6 mm. The optimal sequences are listed at the bottom of the table, the value Γ - the difference between the lamination parameters of this sequences and the required lamination parameters being only 0.1379. The initial buckling load after optimization is 10.21 kN and the Matlab solution time is 141 seconds on a 4.00 GHz PC.

Table 1: Results for normal case

		ξ_1^A	ξ_2^A	ξ_3^A	ξ_1^D	ξ_2^D	ξ_3^D	ξ_1^B	ξ_2^B	ξ_3^B	h_0/h_i								
After stage 1																			
		0.0918	-0.1280	0.0334	-0.2372	-0.3766	-0.0930	-0.0095	0.0429	-0.0093	0.991								
After stage 2																			
		0.0625	-0.125	0.0625	-0.2158	-0.3887	0.1006	0.0078	0.0469	-0.0234	1								
Ply	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Γ	P_e, kN	Time, s
	90	-45	45	45	45	0	-45	0	-45	0	0	90	90	45	45	-45	0.1379	10.21	141

Table 2 demonstrates the results for the symmetric case, in which the three lamination parameters $\xi_{1,2,3}^B$ were forced to be zero to guarantee symmetric stacking sequences. In this case the final value of Γ is 0.2696 and the solution time is 0.4 second.

Table 2: Results for symmetric case

		ξ_1^A	ξ_2^A	ξ_3^A	ξ_1^D	ξ_2^D	ξ_3^D	ξ_1^B	ξ_2^B	ξ_3^B	h_0/h_i								
After stage 1																			
		0.0431	-0.1143	0.0431	-0.2334	-0.3466	0.075	0	0	0	0.992								
After stage 2																			
		0	0	0	-0.1875	-0.3281	0.0703	0	0	0	1								
Ply	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Γ	P_e, kN	Time, s
	45	90	-45	-45	0	45	90	0	0	90	45	0	-45	-45	90	45	0.2696	10.02	0.4

The balanced case is shown in Table 3. The lamination parameter ξ_3^A is forced to be zero to achieve the balanced layup. The final value of Γ is 0.367, solution time is 10 seconds.

Table 3: Results for balanced case

		ξ_1^A	ξ_2^A	ξ_3^A	ξ_1^D	ξ_2^D	ξ_3^D	ξ_1^B	ξ_2^B	ξ_3^B	h_0/h_i								
After stage 1																			
		0.0349	-0.1526	0	-0.2458	-0.3199	0.1188	0.0152	0.0805	0.0552	0.994								
After stage 2																			
		0	-0.25	0	-0.2344	-0.3086	0.0586	0.0156	-0.0625	0.0469	1								
Ply	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Γ	P_e, kN	Time, s
	90	-45	45	45	0	-45	-45	90	0	45	0	45	-45	-45	90	45	0.367	10.13	10

When the stacking sequence is required to be symmetric and balanced, the four lamination parameters ξ_3^A , $\xi_{1,2,3}^B$, must be zero during the two-stages of the optimization. Table 4 illustrates the results of this case in which $\Gamma = 0.244$ and the solution time for the second stage is 0.15 seconds.

Table 4: Results for symmetric and balanced case

		ξ_1^A	ξ_2^A	ξ_3^A	ξ_1^D	ξ_2^D	ξ_3^D	ξ_1^B	ξ_2^B	ξ_3^B	h_0/h_i								
After stage 1																			
		0.0383	-0.1099	0	-0.2422	-0.3512	0.0842	0	0	0	0.991								
After stage 2																			
		0	0	0	-0.2813	-0.375	0.1172	0	0	0	1								
Ply	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Γ	P_e, kN	Time, s
	45	-45	90	90	45	-45	0	0	0	0	-45	45	90	90	-45	45	0.244	10	0.15

The values of Γ in these four different cases are all close to zero illustrating the excellent performance of the Matlab program. All of the final sequences given by the second stage can carry the design load. The time to complete the optimization for these examples is short, especially when the layup is forced to be symmetric and balanced, showing the ability of the method to reduce the search time.

4. Conclusions

A new two-stage method to perform global layup optimization of laminated composite plates with initial buckling constraints has been presented. In the first stage, lamination parameters are used as design variables in VICONOPT to find the optimal plate thickness and required lamination parameters for stage two. The Matlab program used in the second stage employs the branch and bound method and a layerwise technique to find the optimal stacking sequences which can be forced to be symmetric, balanced or both. The results of four examples with different requirements presented in this paper show the versatility of this two-stage method, with the time taken for this global optimization illustrating its efficiency.

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