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NUMERICAL STUDY OF CONVECTIVE HEAT AND MASS TRANSFER THROUGH A SATURATED POROUS MEDIUM IN HORIZONTAL CYLINDRICAL ANNULUS

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ABSTRACT

Two-dimensional heat and mass transfer of natural convection in an annular cylindrical space filled with fluid-saturated porous medium, is analyzed by solving numerically the mass balance, momentum, energy and concentration equations, using Darcy's law and Boussinesq approximation. Both walls delimiting the annular space are maintained at two uniform different temperatures and concentrations. The external parameter considered is Rayleigh-Darcy number. For the present work, the heat and mass transfer for natural convection is studied for the case of aiding equal buoyancies, where the flow is generated in a cooperative mode by both temperature and solutal gradients. The local Nusselt and Sherwood numbers are presented in term of the external parameter.

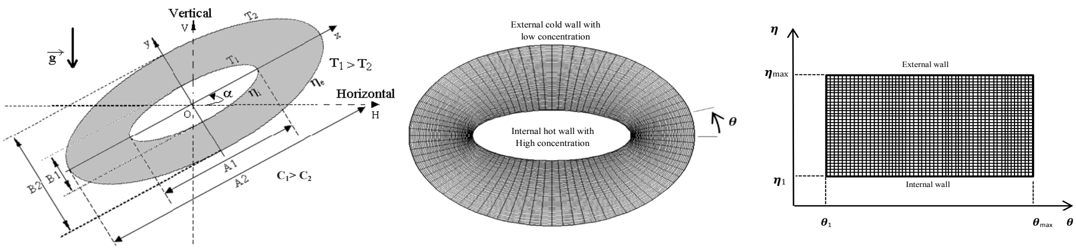
Keywords: *Heat and mass transfer; Natural convection; Porous media; cylindrical annulus.*

1. Introduction

The heat and mass transfer of natural convection confined into different annular enclosures was a subject of many theoretical, numerical and experimental studies. These annular spaces have different geometries and can be partly or completely filled with porous media. Interest in this phenomena is due to many potential applications in the engineering processes which involve the oil and gas industries, thermal recovery process, the underground spreading of chemical waste and other pollutants…etc.
F.M. Mahfouz [1] has investigated a buoyancy driven flow and associated heat convection in an elliptical enclosure. N. Allouache and al. [2] analyzed a solid adsorption refrigerator using activated carbon/methanol pair. It is a contribution to technology development of solar cooling systems. Edimilson J and al. [3] examined a numerical computation for laminar and turbulent natural convection within a horizontal cylindrical annulus filled with a fluid saturated porous medium. Leong and Lai [4] presented a natural convection in concentric cylinders with a porous sleeve, analytical solutions obtained through perturbation method and Fourier transform. Mota and al. [5] solved the two-dimensional Darcy-Boussinesq equations, governing natural convection heat transfer in a saturated porous medium, in generalized orthogonal coordinates, using high-order compact finites differences on a very fine grid.

2. Problem Formulation And Basic Equation

We consider a thermosolutale natural convection in an annular elliptical space filled with fluid-saturated porous medium. Figure 1(a) represents a cross section of the system. Both elliptic internal and external walls are isothermal and impermeable, kept at constant temperatures and concentrations T_1, C_1 and T_2, C_2 respectively with $T_1>T_2$ and $C_1>C_2$. The physical properties of the fluid are constant, apart from the density ρ whose variations are at the origin of the natural convection. Viscous dissipation and the radiation are neglected, Soret and Dufour effects are also neglected and we admit that the problem is bidimensional, permanent and laminar and the porous medium is considered isotropic and homogeneous. Dimensionless equations within the framework of the Boussinesq's approximation after the transformation from the Cartesian to the elliptic coordinates are:



(a) (b)

Figure 1: (a) The cross section of the system, (b) Physical domain (left) and computational domain (right)

Continuity Equation:

$$\frac{\partial}{\partial\eta}\left(HV_{\eta}^{+}\right)+\frac{\partial}{\partial\theta}\left(HV_{\theta}^{+}\right)=0 \tag{1}$$

Momentum Equation:

$$\frac{1}{h}\left[\frac{\partial^2\psi^+}{\partial\eta^2}+\frac{\partial^2\psi^+}{\partial\theta^2}\right]=-Ra_m.H\left(\left[\cos(\alpha)F(\eta,\theta)-\sin(\alpha)G(\eta,\theta)\right]\left(\frac{\partial T^+}{\partial\eta}+N\frac{\partial C^+}{\partial\eta}\right)-\left[\sin(\alpha)F(\eta,\theta)+\cos(\alpha)G(\eta,\theta)\right]\left(\frac{\partial T^+}{\partial\theta}+N\frac{\partial C^+}{\partial\theta}\right)\right) \tag{2}$$

Heat Equation:

$$HV_{\eta}^{+}\frac{\partial T^+}{\partial\eta}+HV_{\theta}^{+}\frac{\partial T^+}{\partial\theta}=\left[\frac{\partial^2T^+}{\partial\eta^2}+\frac{\partial^2T^+}{\partial\theta^2}\right] \tag{3}$$

Concentration Equation:

$$HV_{\eta}^{+}\frac{\partial C^+}{\partial\eta}+HV_{\theta}^{+}\frac{\partial C^+}{\partial\theta}=\frac{1}{Le}\left[\frac{\partial^2C^+}{\partial\eta^2}+\frac{\partial^2C^+}{\partial\theta^2}\right] \tag{4}$$

V_η and V_θ are the velocity components in the directions η and θ, $F(\eta,\theta)$, $G(\eta,\theta)$ used in (2) are the coefficients resulting from the transformation and H represent the metric coefficients in the elliptic coordinates. Ra_m represents Rayleigh-Darcy number which is defined as: $Ra_m=Ra.Da$

The boundary conditions are expressed as following:

Hot inner wall with high concentration ($\eta=\eta_i=cst$):

$$V_{\eta}^{+}=V_{\theta}^{+}=\frac{\partial\psi^+}{\partial\eta}=\frac{\partial\psi^+}{\partial\theta}=0\,,\ T_1^+=1,\ C_1^+=1$$

Cold outer wall with low concentration ($\eta=\eta_e=cst$):

$$V_{\eta}^{+}=V_{\theta}^{+}=\frac{\partial\psi^+}{\partial\eta}=\frac{\partial\psi^+}{\partial\theta}=0\,,\ T_2^+=0,\ C_2^+=0$$

3. Numerical method

Figure 1 (b) shows the physical and computational domain and to solve (1), (3) and (4) with the associated boundary conditions; we consider a numerical solution by the finite volumes method, exposed by [6]. The power law scheme was used for the discretization. To solve (2), we consider a numerical solution by the centred differences method. The iterative method used for the numerical solution of algebraic system of equations is the Gauss-Seidel with an under-relaxation process. Local Nusselt and Sherwood numbers in the physical domain are defined as:

$$Nu=-\frac{1}{h}\frac{\partial T^+}{\partial\eta}\bigg|_{\eta=cst}\ ,\ Sh=-\frac{1}{h}\frac{\partial C^+}{\partial\eta}\bigg|_{\eta=cst}$$

4. Results and discussion

The objective is to analyze the effect of Rayleigh-Darcy number for the case of a cooperative mode of the heat and mass transfer. For this reason, we presented streamlines, isotherms and concentration contours for different values of Rayleigh-Darcy number for the case when the buoyancy ratio N=1 and for a determined value of Lewis number Le=0.1. The Nusselt and Sherwood numbers are presented for different values of Ra_m. The study was carried out for the case of the air and when the eccentricities of the internal and the external ellipses are respectively given by e_1=0.14 and e_2=0.07 in order to obtain a cylindrical configuration and the inclination of the system is α=0°. Figure 2 represent the streamlines (left), concentration contours (middle) isotherms (right), we note that these contours are symmetrical about the median fictitious vertical plane. The streamlines of figure 2 show that the flow is organized in two main cells that rotate in opposite directions. This is due to upward movement of the fluid particles under the aiding buoyancy effect related to temperature and solutal gradients, the fluid heat up along the hot wall and the downward movements of the fluid particles which cool along the cold wall under the gravity. Isotherms in figure 2(a) deform in the upper space where there is presence of two counter-rotating vortices. This configuration illustrates that the heat transfer is dominated by a convective mode in the upper space, in the lower space the heat transfer is dominated by a conduction mode with a slight contribution of the convection.

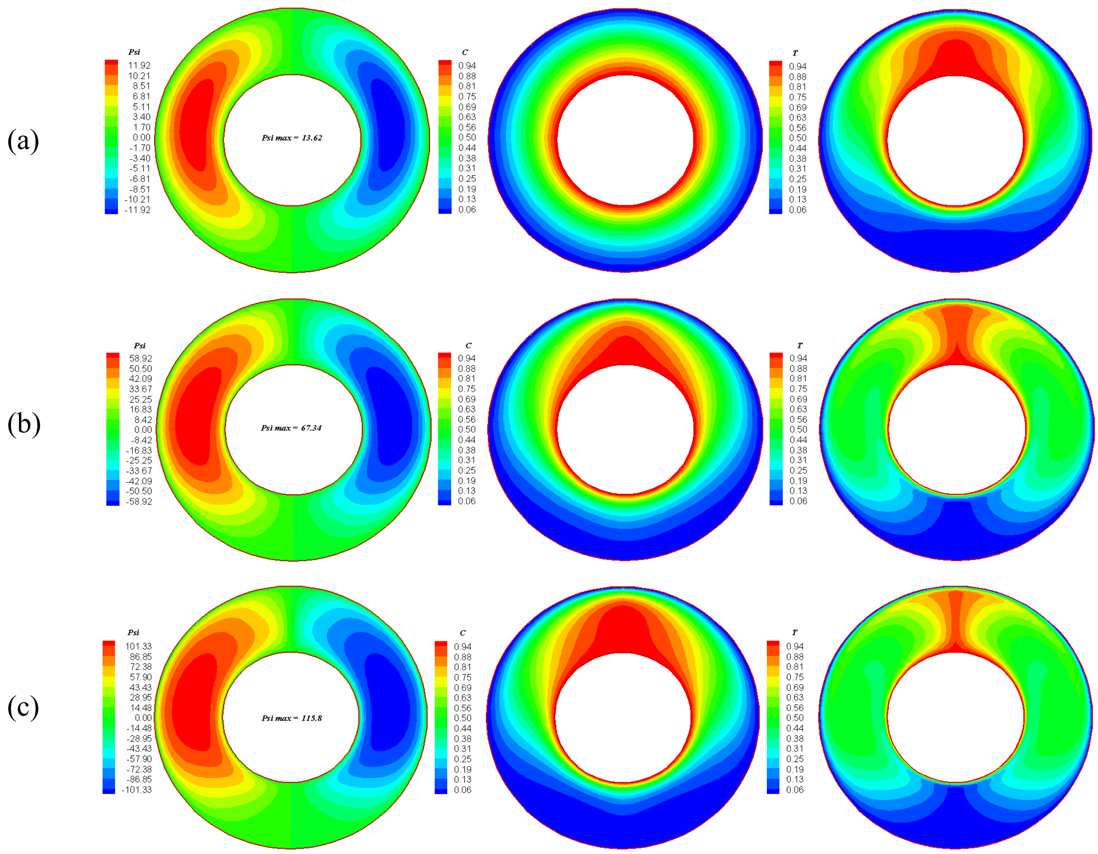


Figure 2: Streamlines, isotherms and concentration contours for N=1 and Le=0.1
(a) Ra_m=10, (b) Ra_m=100 and (c) Ra_m=250

The concentration contours in this figure are parallel and concentric closed curves which coincide perfectly with the walls profile in the annular space where the mass transfer is purely conducted by diffusion mode. In figure 2(b) we notice that with the increase isotherms show that the convective mode is gaining more space in the bottom section under the effect of the thermal gradient. The concentration contours show that the mass transfer is driven by a diffusive mode with a slight transition in the upper space due to the aiding buoyancies. Figure 2(c) shows that the streamlines from both cells tend to become adjacent which decrease the gap between the cells in the upper annulus space, the flow remains organized in two main cells rotating in opposite directions with very high motion. In the same figure, isotherms have a significant change and are increasingly distorted at the top of the annular space due to the increase of the thermal gradient. The concentration contours illustrate a solutal stratification relatively decreasing in the upper section of the annular space.

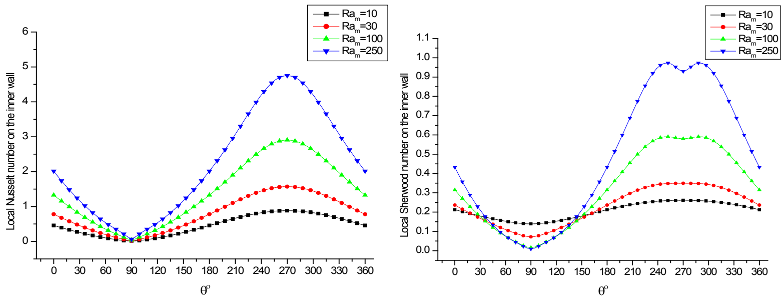


Figure 3: Variation of local Nusselt and Sherwood numbers on the inner wall for *Le*=0.1and *N*=1

Figure 3 illustrates the variation of local Nusselt and Sherwood numbers on the inner wall of the cylinder in term of Rayleigh-Darcy number. For the local Nusselt number this variation allows us to note that with the increase of Rayleigh-Darcy number, the value of local Nusselt number increase significantly due to the increase in the thermal gradient which is obvious. The local Sherwood number increases with increasing of Ra_m due to the aiding effect of thermal and solutal buoyancies.

5. Conclusion

Heat and mass transfer of natural convection in a porous cylindrical annulus saturated by a Newtonian fluid was studied by a numerical method using the method of finite volumes and the vorticity-streamline formulation. We examined, in particular, the influence of Rayleigh-Darcy number for the case when the thermal and the solutal buoyancies are equal and cooperating in the generation of the flow. The structures of bicellular convection take place according to the value of the Rayleigh-Darcy number. When increasing Ra_m the heat transfer is dominated by the convective mode in the entire annular space. The mass transfer remains dominated by a diffusive mode due to the high solutal diffusivity which is ten times higher than the thermal diffusivity for Le=0.1. With the increase of Ra_m the convection mode of the mass transfer rises in the upper space as consequence of the aiding effect of the thermal and the solutal buoyancies when the buoyancy ratio *N*=1.

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