Proceedings of the 24th UK Conference of the
Association for Computational Mechanics in Engineering
31 March – 01 April 2016, Cardiff University, Cardiff

MODELLING FLUID FLOWS THROUGH MULTIPLE DISCRETE FRACTURES USING FINITE ELEMENTS FOR GROUND ENERGY PROBLEMS

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ABSTRACT

Many geo-energy and geo-environmental problems require reliable and accurate predictions of fluid flow through fractured/faulted media. Examples of such problems include the extraction of methane from shale, the sequestration of carbon dioxide in coal beds and the circulation of water through faulted rock as part of an enhanced geothermal system (EGS). Computational modelling is important for the systematic assessment of the feasibility, reliability and performance of these technologies. Central to this computational modelling are the procedures used to account for the fluid flow through the fractures and its transfer in/out of the surrounding bulk material. In the following, a computational approach to modelling fluid flow through fractures is summarised, with an example simulation considered to demonstrate the influence of fracture aperture on the flow characteristics.

Keywords: *fracture; flow; discrete; finite elements; geo-energy*

1. Introduction

Based the work of Thomas and He [1], and following many subsequent developments, the computational code, COMPASS, has been developed at Cardiff University for the assessment of transport phenomena in deformable partially saturated media. COMPASS assumes C_0 continuity in the spatial domain via the use of finite elements and makes use of a finite difference scheme to discretise the underlying equations in time. While previous work has been undertaken using the code to model discretely fractured media [2, 3], this involved meshing both the interior of the fractures and the surrounding continuum. This approach is appropriate for problems involving a small number (<10) of large faults, but for more complex fracture networks it is impractical owing to the need to use a highly refined mesh for each fracture and the consequential high computational cost. Further, it does not lead naturally to an extension to consider fracture propagation.

An alternative approach is to consider the fractures as entities of one spatial dimension less than that of the overall continuum (i.e. as one-dimensional entities in a two-dimensional continuum and two-dimensional entities in a three-dimensional continuum), with the flow considered to be uniform across the aperture of the fracture. In the following, developments undertaken to implement such an approach to discrete fracture flow modelling in a finite element context are summarised.

2. Governing Equations, Fracture and Bulk Flow Coupling and Discretisation

The equations governing flow through the bulk material are those of continuity and Darcy's law, given respectively (using tensor notation) as:

$$\frac{\partial(\rho v_i)}{\partial x_i} + \rho q_{bulk} + \frac{\partial(\rho S n)}{\partial t} = 0 \tag{1}$$

$$v_i = -\frac{k_{ij}}{\mu}\frac{\partial p}{\partial x_j} \tag{2}$$

where ρ is the fluid density, v_i is the fluid velocity (in the global Cartesian coordinates, x_i), q_{bulk} is the fluid flux per unit volume, S is the degree of saturation, n is the porosity, k_{ij} is the intrinsic permeability tensor, μ is the isotropic (scalar) viscosity and p is the fluid pressure. The fully saturated case is considered here; $S = 1$. Applying Galerkin's weighted residual method and introducing a finite element basis gives the following discrete form of the bulk transport equations:

$$\bar{F}_a = \bar{K}_{ab} P_b \tag{3}$$

where:

$$\bar{K}_{ab} = \int_{\Omega_\mathrm{b}} \frac{\partial N^b}{\partial x_j} \frac{\rho k_{ij}}{\mu} \frac{\partial N^a}{\partial x_i} d\Omega \tag{4}$$

$$\bar{F}_a = -\int_{\Gamma_\mathrm{ext,b}} \rho v_{ne} N_a \, d\Gamma + \int_{\Gamma_\mathrm{f}} \rho [\![v_n]\!] N_a \, d\Gamma - \int_{\Omega_b} \rho q_{bulk} N_a d\Omega - \int_{\Omega_\mathrm{b}} \frac{\partial (n\rho)}{\partial t} N_a d\Omega \tag{5}$$

Here, Ω_b is the spatial domain of the bulk material, $\Gamma_\mathrm{ext,b}$ is the external boundary of Ω_b, Γ_f is the boundary of a fracture within Ω_b, P_i are the nodal pressures, N_i are the shape functions, v_{ne} is the normal component of outflow on $\Gamma_\mathrm{ext,b}$ and $[\![v_n]\!]$ is the difference in the normal component of velocity across the fracture. Similar equations govern the flow through the fractures:

$$\frac{\partial (\rho \tilde{v}_i)}{\partial \tilde{x}_i} + \frac{\partial \rho}{\partial t} = 0 \tag{6}$$

$$\tilde{v}_i = -\tilde{k}_{ij} \frac{\partial p}{\partial \tilde{x}_j} \tag{7}$$

where $\tilde{v}_i$ is the fluid velocity in the local Cartesian coordinates, $\tilde{x}_i$, and $\tilde{k}_{ij}$ is the local permeability tensor. For a problem in two dimensions $(i, j = 1,2)$ with the $\tilde{x}_1$ axis tangential to the fracture, the restriction to one-dimensional flow through the fracture is succinctly made by setting the $\tilde{k}_{ij} = {b^2}/{12\mu}$ for $i = j = 1$, where b is the fracture aperture (in accordance with lubrication theory) and $\tilde{k}_{ij} = 0$ otherwise. Again, applying Galerkin's weighted residual method and introducing a finite element basis gives:

$$F^a = K_{ab} P^b \tag{8}$$

where:

$$K_{ab} = \int_{\Omega_\mathrm{f}} \frac{\partial N^a}{\partial \tilde{x}_i} \rho \tilde{k}_{ij} \frac{\partial N^b}{\partial \tilde{x}_j} d\Omega \tag{9}$$

$$F^a = -\int_{\Gamma_\mathrm{f}} \rho [\![v_n]\!] N_a \, d\Gamma - \int_{\Omega_\mathrm{f}} \frac{\partial \rho}{\partial t} N^a d\Omega \tag{10}$$

Here, Ω_f is the spatial domain of the fracture (an area in two-dimensions, a volume in three-dimensions). Owing the restriction placed on the form of $\tilde{k}_{ij}$, the integrals over the spatial domain of the fracture, Ω_f, reduce to integration over Γ_f, so that for a two-dimensional problem:

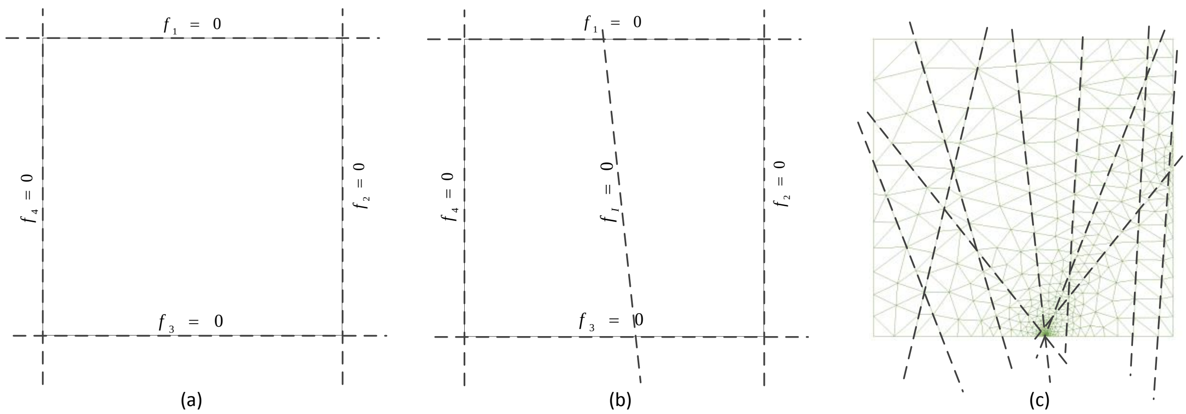
$$K_{ab} = \int_{\Gamma_\mathrm{f}} \frac{\partial N^a}{\partial \tilde{x}_1} \rho \tilde{k}_{11} \frac{\partial N^b}{\partial \tilde{x}_1} b d\Gamma \tag{11}$$

$$F^a = -\int_{\Gamma_\mathrm{f}} \rho [\![v_n]\!] N_a \, d\Gamma - \int_{\Gamma_\mathrm{f}} \frac{\partial \rho}{\partial t} b \, N^a d\Gamma \tag{12}$$

Elimination of the term: $\int_{\Gamma_\mathrm{f}} \rho [\![v_n]\!] N_a \, d\Gamma$, allows the bulk and fracture flows to be readily coupled together. For brevity in the above, flow through the fractures at their intersection with $\Gamma_\mathrm{ext,b}$ has been neglected.

3. Mesh Generation

The approach considered here does not involve enrichment of the finite element basis to allow fractures to bisect elements (as is the case in X-FEM, for example). Instead, the fractures are required to align to element boundaries. It is often reported that generating meshes to align to internal fracture boundaries is problematic (*e.g.* [4]). As such, we elaborate briefly on our approach to mesh generation. Figure 1a displays an initial two-dimensional domain, the boundary of which is succinctly defined by the function $f_\alpha = 0$, where $f_\alpha = max(f_1 \dots f_N)$ and $f_1 = 0, \dots f_N = 0$ are functions defining the lines (in two dimensions) or planes (in three dimensions) that form the boundaries of the domain. The introduction of a fracture (Figure 1b), defined by the function $f_I = 0$, subdivides the initial domain into two subdomains. These two subdomains are succinctly defined by augmenting the definition of f_α to $max(f_1 \dots f_N, f_I) = 0$ and introducing the second function: $f_\beta = max(f_1 \dots f_N, -f_I) = 0$. Repeating this approach for all fractures gives M functions of the form $f_\alpha = 0, f_\beta = 0 \dots f_M = 0$, each defining one subdomain formed from the intersections of all the fractures cutting the domain. With the subdomains defined in this manner, it is a straightforward task to deduce the data needed to pass to third-party mesh generation software to obtain the mesh itself. For example, the software GiD [5] has been used here to obtain the mesh shown in Figure 1c. This approach can also readily be used in three-dimensions.



(a) (b) (c)

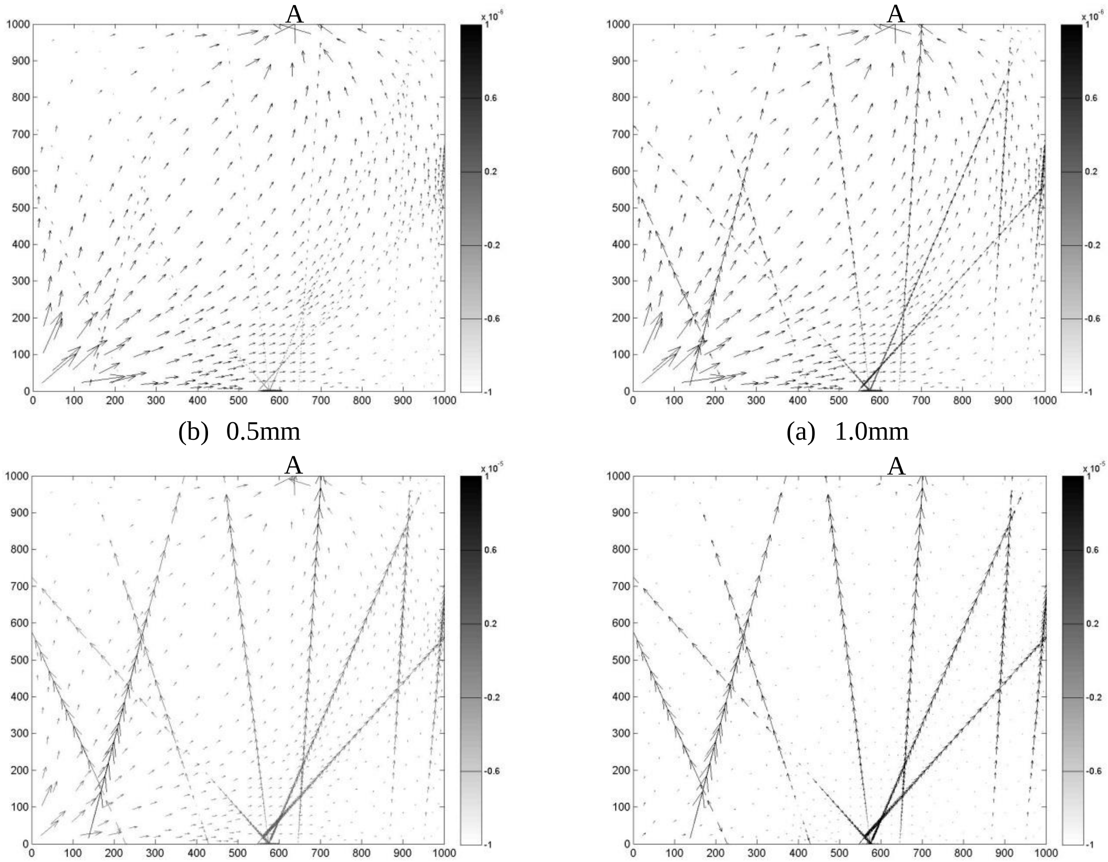
Figure 1: Mesh generation strategy

4. Simulation Results

The finite element formulation described in the previous sections has been implemented in a numerical code. Sample simulation results using this code are shown in Figure 2. The sample problem consists of a two-dimensional (plane strain) square domain of 1km side length. The mesh shown in Figure 1c was used for this simulation, together with the fracture network indicated by the dashed lines on this figure. The intrinsic permeability of the bulk material is 10^{-7}m^2 and the viscosity of the fluid is 10^{-3}Pas with four values for the fracture aperture considered (0.5mm, 1mm, 2mm and 5mm). Fluid is injected at a rate of $0.001\text{m}^2/\text{s}$ at the origin with outflow permitted at the point labelled, A. Velocity vectors obtained after 1 hour of flow through the domain are shown in the figure (the velocity scale of the plots is m/s). Comparison between the figures reveals that for the 0.5mm case (Figure 2a) the flow occurs principally through the bulk, with very minimal preferential flow through the fracture network. For the 1mm case (Figure 2b), the flow through the fracture is more pronounced, while flow through the fractures dominates for the 2mm case (Figure 2c). For the 5mm case (Figure 2d), the flow is almost exclusively through the fracture network with negligible flow through the bulk material. This sample simulation therefore provides an indication of the relative importance of fracture aperture on the flow characteristics of a given fractured rock mass.

5. Conclusions and further work

A finite element formulation for flow through a fractured body is described, accounting for the fluid exchange with the bulk material. Extensions are currently underway to account for the multi-component unsaturated case, and to analyse convective flows. These extensions build upon the modelling framework outlined here.



(b) 0.5mm (a) 1.0mm

(d) 2.0mm (c) 5.0mm

Figure 2: Velocity vectors for various fracture apertures

Acknowledgements

The work described here has been carried out as part of the GRC's Seren project, which is funded by the Welsh European Funding Office (WEFO). The financial support is gratefully acknowledged.

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