

SMOOTH STATIC AND DYNAMIC CRACK PROPAGATION

*Chris J. Pearce¹, Lukasz Kaczmarczyk¹

¹School of Engineering, University of Glasgow, G12 8QQ

*Chris.Pearce@glasgow.ac.uk

ABSTRACT

This paper presents both the theoretical basis for simulating unstable crack propagation in 3D hyperelastic continua within the context of configurational mechanics, and the associated numerical implementation. The approach taken is based on the principle of global maximum energy dissipation for elastic solids, with configurational forces determining the direction of crack propagation. The work builds on the developments made by the authors for static analysis, incorporating the influence of the kinetic energy. The nonlinear system of equations is solved in a monolithic manner using a Newton-Raphson scheme. Initial numerical results are presented.

Keywords: *Fracture; Finite element method; ALE; nuclear*

1. Introduction

The numerical simulation of unstable crack propagation in three-dimensional hyperelastic materials is studied within the context of linear elastic fracture mechanics (LEFM) and configurational mechanics [3]. Although quasi-static and dynamic fracture has been widely investigated in continuum mechanics, this remains a challenging topic. Our approach is to develop the physical and mathematical description to determine (a) when a crack will propagate, (b) the direction of propagation and (c) how far/fast the crack will propagate. Furthermore, we require a numerical setting to accurately resolve the evolving displacement discontinuity within the context of the Finite Element Method. In this study, we present a mathematical derivation and numerical implementation that can achieve these goals, solving for conservation of momentum in both the spatial and material domains. The spatial (or physical) domain can be considered as a description of what we physically observe and the material domain is the evolving reference domain due to crack evolution. The theory is an interpretation of linear elastic fracture mechanics and consistent with Griffith's fracture criterion. This paper represents a generalisation of the authors' previous work on static crack propagation, [3]. The approach taken is based on the principle of global maximum energy dissipation for elastic solids, with configurational forces determining the direction of crack propagation. This approach has been successfully adopted by a number of other authors in the context of quasi-static analysis, e.g. [3] and [2]. At present we restrict ourselves to the consideration of elastic bodies with energy dissipation limited to the creation of new crack surfaces.

In the context of the numerical setting, we have adopted the Arbitrary Lagrangian-Eulerian (ALE) method, which is a kinematic framework to describe movement of the nodes of the finite element mesh independently of the material. Thus, we are able to resolve the propagating crack without influence from the original finite element mesh, and maintain mesh quality. The efficient solution of 3D crack propagation, with a large numbers of degrees of freedom, requires the use of an iterative solver for solving the system of algebraic equations. In such cases, controlling element quality enables us to optimise matrix conditioning, thereby increasing the computational efficiency of the solver.

The resulting system of equations is highly nonlinear and requires a solution strategy that can trace the entire transient response. The application of this work is the predictive modelling of crack propagation in nuclear graphite bricks, which are used as the moderator in UK advanced gas-cooled nuclear reactors (AGRs).

2. Kinematics of Propagating Cracks

Differentiable mappings relate the reference material domain to both the current spatial and the current material domains. These mappings are utilized to independently observe the deformation of material in the physical space Ω_t , and the evolution of the crack surface in the material space B_t , see Figure 1.

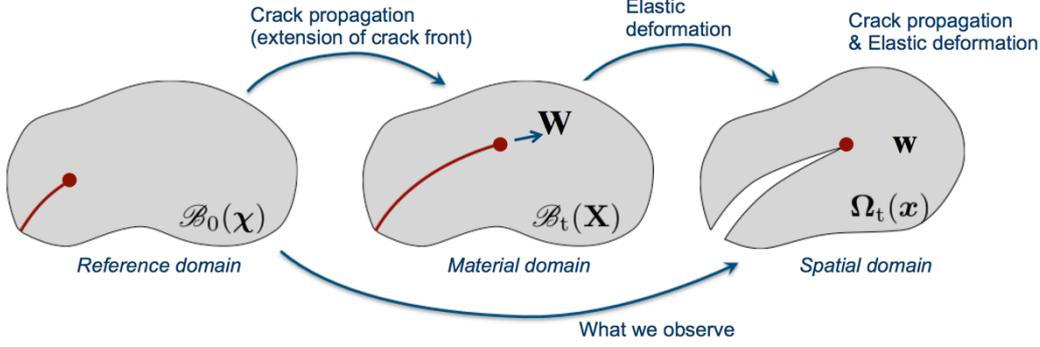


Figure 1: Deformation and configurational change of a body with a propagating crack

$$\mathbf{w} = \mathbf{x} - \boldsymbol{\chi}, \quad \mathbf{W} = \mathbf{X} - \boldsymbol{\chi}, \quad \mathbf{u} = \mathbf{w} - \mathbf{W} \quad (1)$$

3. Numerical Implementation

The finite element approximation is applied to both the physical and material space. Three-dimensional domains are discretised with tetrahedral elements with hierarchical basis functions of arbitrary polynomial order, following the work of [1]. The higher-order approximations are only applied to displacements in the spatial configuration, whereas a linear approximation is used for displacements in the material space. The resulting residuals in the spatial and material domain, that represent the two primary, nonlinear equations that need to be solved, are expressed as:

$$\mathbf{r}_s = \lambda(t) \mathbf{f}_{s,\text{ext}} - \mathbf{f}_{s,\text{int}}, \quad \mathbf{r}_m = \mathbf{f}_{m,\text{res}} - \mathbf{f}_{m,\text{driv}} \quad (2)$$

where λ is the load factor that scales the external reference load, $\mathbf{f}_{s,\text{ext}}$; $\mathbf{f}_{s,\text{int}}$ is the internal force vector, $\mathbf{f}_{m,\text{res}}$ is the material resistance and $\mathbf{f}_{m,\text{driv}}$ is the driving force for crack propagation. These equations are linearised and solved using a Newton-Raphson procedure.

The spatial internal force vector is expressed as

$$\mathbf{f}_{s,\text{int}} = \int \mathbf{B}_X^T \mathbf{P} \, dV + \int \rho \mathbf{N}^T \mathbf{a} \, dV \quad (3)$$

and the material driving force is expressed as

$$\mathbf{f}_{m,\text{driv}} = \int \mathbf{B}_X^T \boldsymbol{\Sigma} \, dV - \int \rho_0 (\mathbf{F}^T \mathbf{a} + \dot{\mathbf{F}}^T \mathbf{v}) \, dV = \mathbf{G} - \int \rho_0 (\mathbf{F}^T \mathbf{a} + \dot{\mathbf{F}}^T \mathbf{v}) \, dV \quad (4)$$

where \mathbf{P} is the first Piola Kirchoff stress tensor, $\boldsymbol{\Sigma}$ is the Eshelby stress, \mathbf{a} is the spatial acceleration, $\mathbf{v} = \dot{\mathbf{u}} = \dot{\mathbf{w}} - \dot{\mathbf{F}}\mathbf{W}$ is the spatial velocity, \mathbf{F} is the deformation gradient and \mathbf{G} is the configurational force. The Eshelby stress is defined as:

$$\boldsymbol{\Sigma} = W\mathbf{I} - \mathbf{F}^T \mathbf{P} \quad (5)$$

where W is the strain energy density.

In the restricted case of quasi-static analysis, the inertia and velocity terms in (3) and (4) vanish and the formulation reverts to that presented in [3]. Thus, equations (3) and (4) represent an important development in the modelling of dynamic crack propagation, generalising the configurational mechanics formulation. It is important to emphasise that, for fast crack propagation, whereby the inertia terms in equations (5) and (6) are included, the crack front velocities are calculated to satisfy

equilibrium of the crack front in the material space. Therefore, the crack front velocity is not a material or model parameter, as is the case in most models, but a natural result of a consistent mathematical derivation, starting from the first law of thermodynamics [3].

4. Mesh Quality

The continuous adaptation of the finite element mesh to resolve the propagating crack will result in a degeneration of the mesh quality. For large problems, where it is necessary to use iterative solvers, we need to control the quality of the elements to ensure good matrix conditioning. The key challenge is to enforce constraints to preserve element quality for each Newton-Raphson iteration, without influencing the physical response. Thus, we introduce a measure of element quality for tetrahedral elements in terms of their shape and use this to drive mesh improvement. Here we include both node movement and changes in element topology (face flipping). Thus, equations (4) and (5) are augmented by a third residual, \mathbf{f}_q , defined as

$$\mathbf{f}_q = \int \mathbf{B}_x^T \mathbf{Q} dV \quad (6)$$

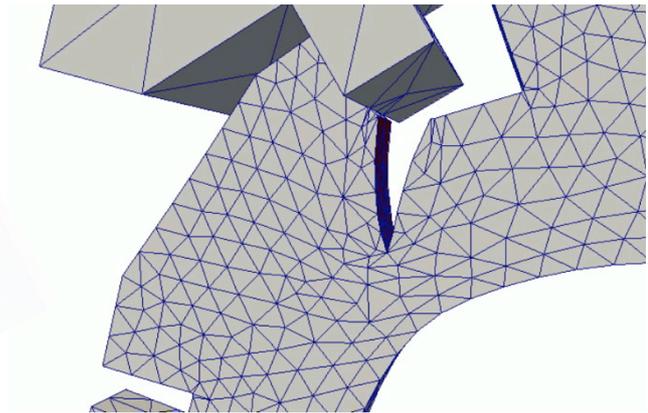
where \mathbf{Q} is a pseudo “stress” at the element level, as a counterpart to the first Piola Kirchhoff stress. Since the conditioning of the finite element stiffness matrix is controlled by the quality of the worst elements, we advocate that \mathbf{Q} is a function of a log-barrier objective function as a means of evaluating the quality of an entire mesh (whilst punishing harshly the worst quality element), and the volume-length quality measure, [4].

5. Numerical Examples

To demonstrate the performance of the model for quasi-static loading, a numerical example is presented which considers a slice of a graphite brick under quasi-static loading of the keyway. Figure 3 shows a good comparison between the numerically simulated crack path and the experimentally observed crack. Figure 3(d) shows that the results are independent of the mesh size.



(a)



(b)

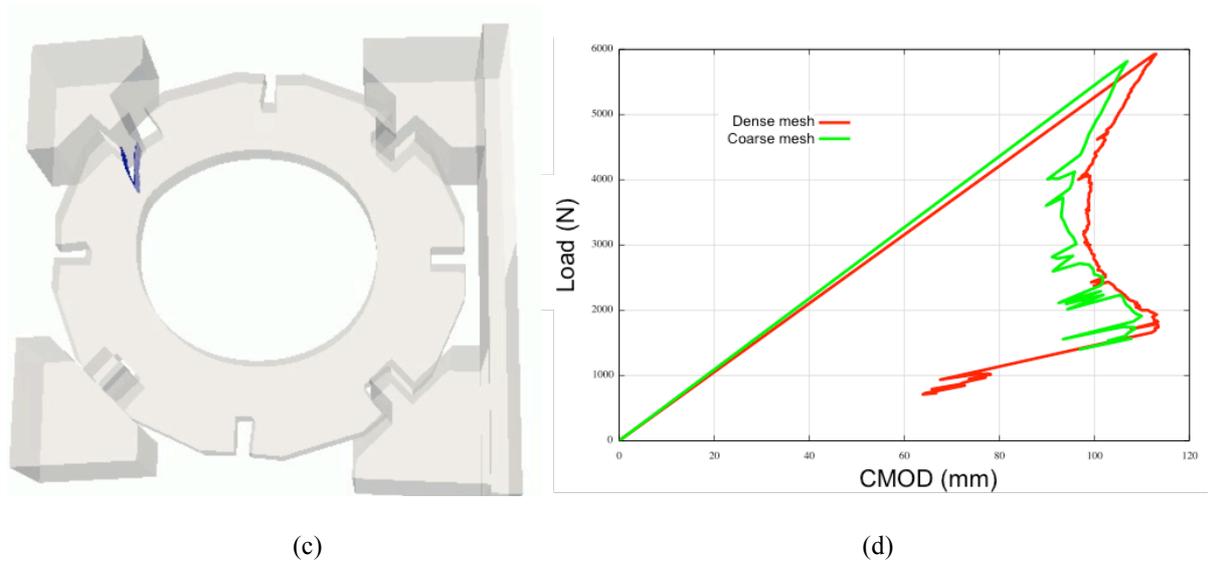


Figure 3: Graphite brick slice. (a) Experimentally observed crack; (b) numerically predicted crack; (c) model geometry and predicted crack; (d) load-displacement response.

6. Conclusions

In this study, the basis for unstable crack propagation using configurational mechanics has been presented. The highly nonlinear system of equations are implemented in and solved using MoFEM, a finite element code for multi-physics problems which is developed at the University of Glasgow. Performance of the model is demonstrated on three numerical problems. The implementation has proved to be stable, robust and computationally efficient.

Acknowledgements

This work was supported by EDF Energy Nuclear Generation Ltd and the Royal Academy of Engineering (RAEng). The views expressed in this paper are those of the authors and not necessarily those of EDF Energy Nuclear Generation Ltd or RAEng.

References

- [1] Ainsworth, M.A. and Coyle, J. Hierarchic finite element bases on unstructured tetrahedral meshes. *International Journal for Numerical Methods in Engineering*. 58, 2103–2130, 2003.
- [2] Gurses, E. and Miehe, C. A computational framework of three-dimensional configurational-force-driven brittle crack propagation. *Computer Methods in Applied Mechanics and Engineering*, 198, 1413-1428, 2009.
- [3] Kaczmarczyk, Ł., Mousavi, M. and Pearce, C.J. Three-dimensional brittle fracture: configurational-force-driven crack propagation. *International Journal for Numerical Methods in Engineering*, 7, 531–550, 2014.
- [4] Pearce, C.J., MacKenzie, R. and Kaczmarczyk, Ł. Numerical approaches for brittle crack propagation, in B.H.V. Topping, (Eds), *Computational Methods for Engineering Science*, Saxe-Coburg Publications, Stirlingshire, UK, Chapter 6, pp 135-155, 2014.