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EFFICIENT IMPLEMENTATION OF VOLUME/SURFACE INTEGRATED AVERAGE BASED MULTI-MOMENT METHOD

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ABSTRACT

We investigate discretization strategies of the conservation equation in VSIAM3 (volume/surface integrated average based multi-moment method) which is a numerical framework for incompressible and compressible flows based on a multi-moment concept. We investigate these strategies through the droplet splashing on a superhydrophobic substrate. We find that the use of the CIP-CSLR (constrained interpolation profile-conservative semi-Lagrangian with rational function) method as the conservation equation solver is critically important for the robustness of incompressible flow simulations using VSIAM3 and that numerical results are sensitive to discretization techniques of the divergence term in the conservation equation.

Key Words: *multi-moment method; VSIAM3; CIP-CSL method; droplet splashing*

1. Introduction

VSIAM3 [1, 2] is a numerical framework to simulate fluid flows, and employs a CIP-CSL method [4, 3] as the conservation equation solver. VSIAM3 is a highly robust and efficient numerical framework [5]. However, a multi-moment framework which has been used in VSIAM3 (including the CIP-CSL method) have increased some complexities in the implementation. Although several CIP-CSL methods such as CSL2 (CSL with quadratic function) [4], and CSLR (CSL with rational function) [3] have been proposed, little attention has been given to the formulation of the divergence term in the CIP-CSL methods.

In this study, we propose efficient formulations for the divergence term in the CIP-CSL schemes and we identify reasons for robust implementation of VSIAM3.

2. The CIP-CSL and the Velocity Divergence Term

The CIP-CSL methods are used to solve the conservation equation

$$\frac{\partial}{\partial t}\int_{\Omega}\phi dV + \int_{\Gamma}\phi(\mathbf{u}\cdot\mathbf{n})dS = 0, \tag{1}$$

here ϕ is a scalar value. In the CIP-CSL, an interpolation function $\Phi_i(x)$ is constructed using different moments in one cell. For instance, in the CIP-CSL2 [4] method, a quadratic interpolation function $\Phi_i(x)$

$$\Phi_i(x) = a_i(x - x_{i-1/2})^2 + b_i(x - x_{i-1/2}) + \phi_{i-1/2}, \tag{2}$$

is used to interpolate between $x_{i-1/2}$ and $x_{i+1/2}$ as shown in Fig. 1. By using the following constraints

$$\Phi_i(x_{i+1/2}) = \phi_{i+1/2}, \tag{3}$$

$$\phi_i = \int_{x_{i-1/2}}^{x_{i+1/2}} \Phi_i(x)dx/\Delta x, \tag{4}$$

the coefficients, a_i and b_i, can be determined. In the CIP-CSLR [3] method which is characterised by less numerical oscillations, the following interpolation function

$$\Phi_i(x) = \frac{\alpha_i\beta_i(x - x_{i-1/2})^2 + 2\alpha_i(x - x_{i-1/2}) + \phi_{i-1/2}}{(1 + \beta_i(x - x_{i-1/2}))^2}, \tag{5}$$

with

$$\alpha_i = \beta_i \phi_i + (\phi_i - \phi_{i-1/2})/\Delta x, \tag{6}$$

$$\beta_i = \frac{1}{\Delta x}\left(\frac{|(\phi_{i-1/2} - \phi_i)| + \epsilon}{|(\phi_i - \phi_{i-1/2})| + \epsilon} + 1\right), \tag{7}$$

is used. Here ϵ is an infinitesimal number. Once the interpolation function $\Phi_i(x)$ is ready, the cell average ϕ_i is updated by a finite volume formulation

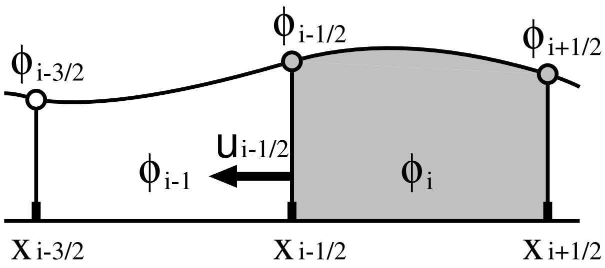

Figure 1: Schematic figure of the CIP-CSL2 method. $u_{i-1/2} < 0$ is assumed. The moments which are indicated by gray color ($\phi_{i-1/2}$, ϕ_i and $\phi_{i+1/2}$) are used to construct the quadratic interpolation function.

$$\frac{\partial}{\partial t}\int_{x_{i-1.2}}^{x_{i+1/2}} \phi dx = -\frac{1}{\Delta x}(F_{i+1/2} - F_{i-1/2}), \tag{8}$$

here $F_{i-1/2}$ is the flux

$$F_{i-1/2} = \begin{cases} -\int_{x_{i-1/2}}^{x_{i-1/2}-u_{i-1/2}\Delta t} \Phi_{i-1}(x)dx & \text{if} \quad u_{i-1/2} \geq 0 \\ -\int_{x_{i-1/2}}^{x_{i-1/2}-u_{i-1/2}\Delta t} \Phi_{i}(x)dx & \text{if} \quad u_{i-1/2} < 0. \end{cases} \tag{9}$$

The boundary value $\phi_{i-1/2}$ can be updated by the conservation equation of a differential form

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = -\phi\frac{\partial u}{\partial x}. \tag{10}$$

(10) is solved using a splitting approach as follows

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0, \tag{11}$$

$$\frac{\partial \phi}{\partial t} = -\phi\frac{\partial u}{\partial x}. \tag{12}$$

A semi-Lagrangian approach is used for the advection equation (11)

$$\phi^*_{i-1/2} = \begin{cases} \Phi_{i-1}(x_{i-1/2} - u_{i-1/2}\Delta t) & \text{if} \quad u_{i-1/2} \geq 0 \\ \Phi_{i}(x_{i-1/2} - u_{i-1/2}\Delta t) & \text{if} \quad u_{i-1/2} < 0. \end{cases} \tag{13}$$

(12) represents a correction due to the divergence term of the velocity and is solved by a finite difference method. We propose the following approximations of the velocity divergence term of the 1D conservation equation.

Simple upwind based on boundary value (UPW)

$$\phi\frac{\partial u}{\partial x} = \begin{cases} \phi^*_{i-1/2}\left(\frac{u^n_{i-1/2} - u^n_{i-3/2}}{\Delta x}\right) & \text{if} \quad u_{i-1/2} > 0 \\ \phi^*_{i-1/2}\left(\frac{u^n_{i+1/2} - u^n_{i-1/2}}{\Delta x}\right) & \text{if} \quad u_{i-1/2} \leq 0. \end{cases} \tag{14}$$

This is a simple upwind approximation based on the boundary values.

Central difference based cell centre value (CDcc)

$$\phi\frac{\partial u}{\partial x} = \phi^*_{i-1/2}\frac{\hat{u}^n_i - \hat{u}^n_{i-1}}{\Delta x} \tag{15}$$

This is a central difference approximation based on the cell center values ($\hat{u}_i$), where $\hat{u}_i$ is the velocity calculated at cell centre [2]

$$\hat{u}_i = \frac{3}{2}u_i - \frac{1}{4}\left(u_{i+1/2} + u_{i-1/2}\right). \tag{16}$$

Mixed formulation of the simple upwind and a central difference (UPW-CDcc)

$$\phi\frac{\partial u}{\partial x} = \begin{cases} \mathrm{D}_{UPW} & \text{if} \quad \mathrm{D}_{UPW}\cdot\mathrm{D}_{CDcc} < 0 \\ \mathrm{D}_{UPW} & \text{else if} \quad |\mathrm{D}_{UPW}| < |\mathrm{D}_{CDcc}| \\ \mathrm{D}_{CDcc} & \text{else,} \end{cases} \tag{17}$$

here D_{UPW} and D_{CDcc} represent $\phi\frac{\partial u}{\partial x}$ which are calculated by (14) and (15), respectively. The mixed formulation is introduced to take advantages of both upwind and central difference approximations.

3. Numerical results of the droplet splashing

We conducted numerical simulations of droplet splashing on a superhydrophobic substrate to study the effects of these discretization strategies of the conservation equation in VSIAM3 through a highly complicated free surface flow problem.For more detail see [5].

In the set of numerical simulations, quantitative parameters, the densities $\rho_{liquid} = 1000$ kg/m^3, $\rho_{air} =$ 1.25 kg/m^3, viscosities $\mu_{liquid} = 1.0\times10^{-3}$ Pa·s, $\mu_{air} = 1.82\times10^{-5}$ Pa·s, surface tension $\sigma = 7.2\times10^{-2}$ N/m, gravity 9.8 m/s^2, initial droplet diameter $D = 1.86$ mm, impact speed 2.98 m/s and the equilibrium contact angle 163° are used. A regular Cartesian grid system of $192\times192\times48$ is used.

Fig. 2 shows the results. VSIAM3 with CSL2-UPW could not capture droplet splashing well as shown in Fig. 2a. CSL2-UPW also was not stable after around 1.1 ms. VSIAM3 with CSL2 with any central difference formulation was not stable for this problem. VSIAM3 with CSLR is stable when UPW was used for the divergence term as shown in Fig. 2b. However if we use any central difference formulation for the divergence term, VSIAM3 with CSLR was also unstable. If we use UPW-CDcc (mixed formulation), VSIAM3 with CSLR could conduct stable numerical simulation of droplet splashing and capture droplet splashing well as shown in Fig. 2c.

4. Conclusions

The numerical results showed that VSIAM3 with CSL2 is not robust enough and that VSIAM3 with CSLR is highly robust (if an appropriate formulation is used for the divergence term). We also found that the numerical results are sensitive depending on discretization formulations of the divergence term in the conservation equation. The numerical results of droplet splashing showed that VSIAM3 with any central difference formulation is not robust even though CSLR is used, while VSIAM3 with the simple upwind formulation was highly robust and captures the droplet splashing well. These results indicates that the use of the upwind formulation is suitable for robust numerical simulations, especially for highly complicated flows like droplet splashing.

We also proposed the mixed formulation using both a central difference and the simple upwind formulation for the divergence term. The mixed formulation can simulate the droplet splashing like the result using the simple upwind. The mixed formulation can take advantages of both central difference and upwind formulations.

In conclusion, employing the less oscillatory CSL scheme (i.e. CSLR) with an appropriate divergence term formulation is critically important for robust implementation of VSIAM3.

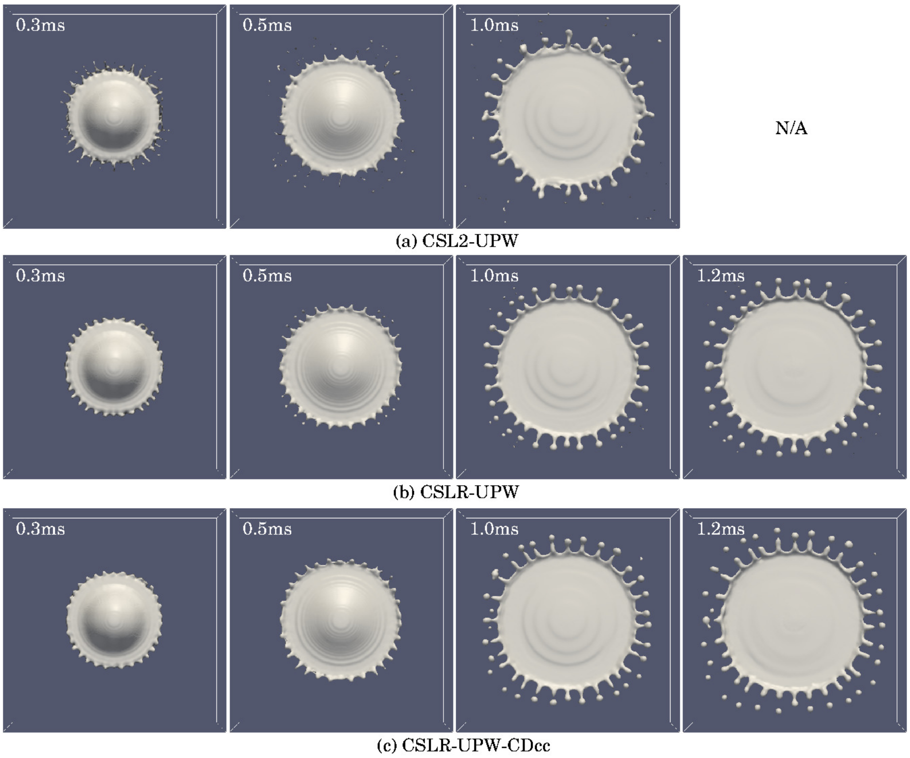

Figure 2: Numerical results of droplet splashing by CSL2-UPW (a), CSLR-UPW (b) and CSLR-UPW-CDcc (c). VSIAM3 with CSL2-UPW was not stable after around 1.1ms.

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