On the Small Scale Nonlinear Finite Element Analysis of Flexible Risers

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ABSTRACT

In this paper an efficient small-scale, detailed finite-element modelling method for flexible risers is presented. The method can be effectively implemented in a fully-nested (FE^2) multiscale analysis based on computational homogenization. To reduce the computational cost only a small fraction of a flexible pipe is used for a detailed nonlinear finite-element analysis at the small scale by exploiting the cyclic symmetry of the model and applying periodic boundary conditions. In this model, using three-dimensional elements, all layer components are individually modelled and a surface-to-surface frictional contact model is used to simulate their interaction. The approach is applied on a 5-layered pipe made of inner, outer and intermediate polymer layers and two intermediate armour layers, each made of 40 steel tendons. The capability of the method in capturing the detailed nonlinear effects and the great advantage in terms of significant CPU time saving are demonstrated by comparing the results obtained on elements of pipe of different lengths.

Keywords: *Periodic Boundaries, Fixed Boundaries, Flexible risers*

1. Introduction

Unbonded flexible risers have become the main means for transporting oil and gas between the seabed and surface in ultra-deep waters. They consist of several polymer and steel layers that can move internally relative to each other. Their ability of withstanding large displacements and rotations makes them ideal for floating platforms. In many problems of very significant industrial interest sufficient accuracy can only be obtained by the use of models that properly take into account contact and friction between layers and how these are related to internal and external pressure. Although FE models can account for the complex internal structure of flexible risers, their computational requirements limit their applicability to just a few meters in length at most. So, a more efficient methodology with lower computational cost is required to bridge the gap between nonlinear dynamic simulations at the large scale and detailed finite element models at the small scale.

One approach to reduce computational cost of the analysis of flexible risers is to develop constitutive laws for large-scale beam models, which link generalised stresses and strains to model the hysteresis loops occurring for flexible pipes subjected to cyclic loading, as shown by Tan et al. [1]. A model based on this approach is used by Alfano et al. [2], building on the analogy between frictional slipping between different layers of a flexible riser and frictional slipping between micro-planes of a continuum medium in non-associative elasto-plasticity. In this way, a linear elastic relationship was used for the initial response, in which no slip occurs, and a non-associative rule with linear kinematic hardening was then introduced to model the full-slip phase. One major challenge in using the constitutive law based on the non-associative elasto-plasticity analogy is the determination of the parameters of the constitutive law to bridge the small scale of the detailed FE simulations with the large scale of the model accurately. An alternative approach which does not have this limitation is a fully-nested multi-scale procedure [3], currently in widespread use for the modelling of composite materials. With this method, at each integration point (i.e. cross section) of the large-scale beam model, the stress resultants corresponding to assigned generalised strains are determined through the solution of the small-scale FE problem. Key to accurate solution of the small scale FE problem is the use of suitable boundary conditions. The

periodic boundary conditions are the most effective and accurate for most cases involving a periodic microstructure or when the microstructure is not periodic but the small scale model is sufficiently statistically representative [4, 5]. The motivation for this work is that previous models in the context of sequential multi-scale analysis [6] are too computationally expensive for a fully nested analysis. Hence, this paper describes an efficient modelling approach for the small-scale analysis, which exploits the cyclic symmetry of the riser detailed structure, and its implementation based on the introduction of periodic boundary conditions for detailed FE models in small scale simulations in a similar method used by [7]. The capability of the boundary condition treatment method in capturing the non-linear effects and the great advantage of significant CPU time saving by this method over using a larger model is shown.

2. Small scale model

A fully-nested computational homogenization scheme is essentially based on the construction of a micro-scale (or more generally small-scale) boundary-value problem (BVP) at each integration point of a macro (or large-scale) model. The small-scale model is solved numerically to determine the constitutive response of the material at each integration point [8]. If the FEM is used at both scales, the methods is also known as FE^2. If a displacement-based FE formulation and a Newton-Raphson incremental scheme are used at the large scale, 'tentative' strains resulting from an attempted displacement increment are calculated at each integration point for each iteration of each increment in the macro model. When a 3D continuum model is used at both scales, the tentative strains are imposed on a suitably defined representative volume element (RVE) of the micro scale (down-scaling procedure), each large-scale integration point corresponding to one and only RVE. For the flexible risers considered in this paper, a 3D continuum model is used at the small scale, which is to be linked to a large-scale beam model, where generalised strains and stress resultants are employed. A simplified 5-layer flexible pipe, made of three polymer layers and two armour layers, was considered. Both the inner and the outer armour layers are made of 40 steel tendons, with rectangular cross section, which are wound with the same pitch of length L_p equal should to 320mm. It is widely accepted that the use of periodic boundary conditions in multiscale computational homogenization provides the most accurate results, at least at sufficient distance from the real boundary of the structure. On the other hand, the solution for a segment of riser whose length is any multiple of L_p/N should be characterized by the same cyclic symmetry if periodic boundary conditions are applied and if such solution is unique. Figure 1 shows the the smallest model which has a length equal to $1/40$ of the pitch length of the tendons. For this small slice of pipe the position of each tendon on one end cross section is the same as the position of the adjacent tendon on the other end cross section. This makes this slice of $1/40$ of length the smallest repeating unit of the pipe. Furthermore, as typical in first-order computational homogenisation, the assumption is made that the variations of the internal stress resultant (i.e. large-scale stress) and of the generalised strain (i.e. large-scale strain) are small enough that, for the element of pipe under consideration, they can be neglected. Therefore, to within a rigid motion, the assumption of periodic kinematics is made, resulting in the enforcement of periodic boundary conditions. A set of linear constraint equations was generated relating the degrees of freedom of each pair of nodes on the boundaries having the same position on the cross-section [9].

3. Numerical results

The study in this paper focuses on cases in which the segments of pipes are subject to bending, as well as internal and external pressure, the latter being balanced to produce a relatively small and outward radial displacement of the inner surface of the inner polymer layer (inner liner). All components are modelled with fully-integrated 8-noded 3D solid elements with incompatible strains, with surface-to-surface frictional contact between all components. Details of the material, dimensions and arrangement of constituent layers are given in Table 1. A fully-implicit nonlinear static analysis based on the Newton-Raphson method is used to solve the models.
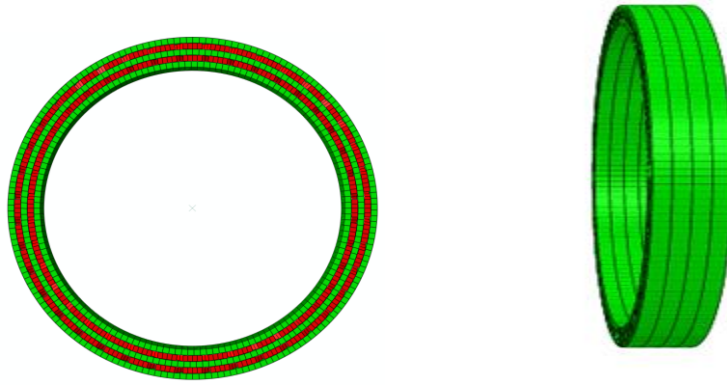
Figure 1: cross section of the FE model and FE models with lengths equal to $L_p/40$

Table 1: Dimensions and materials of components in the model.

Layer	r_0, r_1 (mm)	Material	E (MPa)	υ
1	48, 50	Polyethylene	0.35	0.4
2	50, 52	Carbon Steel	210	0.3
3	52, 54	Polyethylene	0.35	0.4
4	54, 56	Carbon Steel	210	0.3
5	56, 58	Polyethylene	0.35	0.4

The analyses were conducted by applying internal and external pressure in a first step, after which one symmetric cyclic history of bending curvature was prescribed, the maximum and minimum curvatures being 0.125 and -0.125 m^{-1}. The internal and external pressures were equal to 4 and 4.5 MPa, respectively. The curves in Figures 2 show the bending moment against the (prescribed) bending curvature for four models with length equal to L_p/N , with $N = 1, 5, 20, 40$. It can be appreciated that the difference in the results of the models with different lengths is practically negligible when periodic BC is used. These results confirm that use of the smallest repeating unit as the model to be used at the small scale of a multi-scale analysis is a valid choice. The analyses were carried out in parallel on a computer cluster with two dual-core 1.8 GHz processors (32 processors in total) using 8MB of RAM. The huge saving in CPU time, from 11 hours for the longest model of one pitch length to only few minutes, allows the smallest model to be effectively used in a nested multi-scale strategy.
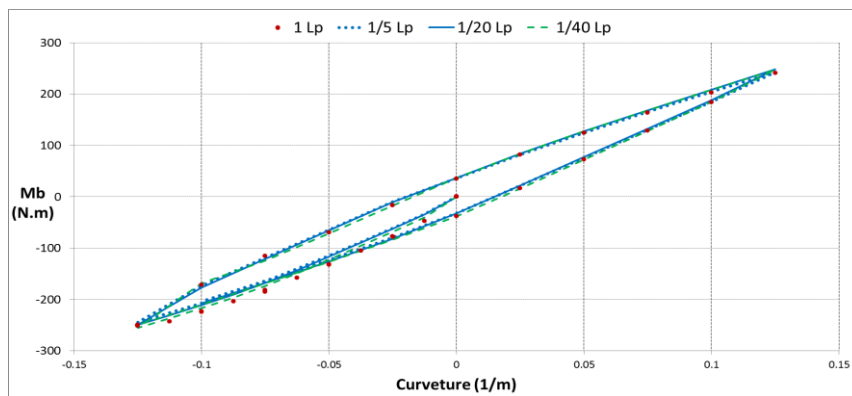

Figure 2: Bending moment vs prescribed bending curvature (internal pressure 4MPa, external pressure 4.5MPa).

4. Conclusions

An effective approach to minimize the computational cost of a detailed nonlinear FE analysis of a segment of flexible riser is presented in this paper. In this method frictional contact between all layers is taken into account. The key ideas behind the proposed approach are (a) the observation that flexible risers can be represented, with very good approximation, with a model having cyclic symmetry, (b) the use of such cyclic symmetry to reduce the length of the model to the smallest repeating unit and (c) the use of periodic boundary conditions. The enormous saving in computational cost entailed by the use of the smallest repeating unit makes this the optimal model to be used in a nested (FE^2) multiscale analysis.

Acknowledgements

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