Numerical investigation of hydraulic fracturing

* D. Mahdavian¹, A.A. Javadi²

¹ Department of Engineering, University of Exeter, North park Road, Exeter, EX4 4QF

*dm464@exeter.ac.uk

ABSTRACT

Hydraulic fracturing is a commonly used method for producing oil and gas from reservoirs. Analysis of hydraulic fracturing requires the study of fracture mechanics and fluids mechanics. The focus of this paper is to investigate the initiation and growth of cracks in a typical well. The finite element method is used to simulate the process of hydraulic fracturing, calculating the effective parameters in fracture analysis and studying the fracture characteristics under different conditions. According to the linear elastic fracture mechanics, if the stress intensity factor is equal to fracture toughness, the crack will propagate. The fracture parameters related to a semi-elliptical crack are studied using the finite element software ANSYS. The numerical results indicate that the stress intensity factor increases by increasing the crack length. The crack length increases by increasing the fluid leakage rate. The relationship between stress intensity factor, leakage rate and crack length (which represents crack growth in each step) is investigated.

Key words: hydraulic fracturing; stress intensity factor; crack; leakage; crack length, finite element

1-Introduction

Hydraulic fracturing is a fracturing processes initiated from a pressurized open borehole section into rock formations. The process is characterized by solid-fluid interaction. On the solid side, the rock formation deforms with the propagation of the fracture front and pressurization of the fracture face. On the other side, the fluid flows at high pressure into the narrow fracture cavity. At the same time, the fluid may also infiltrate into the porous rock. Even in the most basic form, hydraulic fracturing is a complex process, not just because of the heterogeneity of the earth structure, and indeterminate in-situ stresses or rock behaviour, but also because of the physical complexities of the problem. It involves coupling of three processes:

- 1- Mechanical deformation of the formation caused by the pressure inside the fracture,
- 2- Fluid flow within the fracture networks,
- 3- Fracture propagation.

2-Hydraulic fracturing mechanism

Hydraulic fracturing is a 3D phenomenon. Modelling of hydraulic fracturing involves determination of crack width, length and height and distribution of fluid pressure inside the crack as a function of time and location for a known flow rate or known fluid pressure in the borehole. But modelling of this 3 dimensional phenomenon is not easy because hydraulic fracturing includes coupling of at least three processes.

In order to model hydraulic fracturing, it is necessary to solve three governing equations:

- 1) Geo-mechanical behaviour of the reservoir,
- 2) Flow in the porous medium, and
- 3) Flow inside the crack.

The elasticity, fluid flow and fracture growth equations should ideally be fully coupled in order to solve the system of equations correctly and get the crack width, length, height and fluid pressure as a function of time and location. In general, the governing equations of a multiphase porous medium are as follow:

1) Momentum equilibrium equation for the whole multiphase medium,

- 2) Momentum equilibrium equation for each fluid phase,
- 3) Mass conservation equation for each porous medium phase.

Nevertheless, many research works on hydraulic fracturing, assuming elastic behaviour, have used single integral equation as:

$$p(x,y,t) - \sigma_c(x,y) = \int_{\Omega(t)} C(x,y;\xi,\eta) \, w(\xi,\eta,t) \, d\xi \, d\eta$$

or

$$w(x,y,t) = \int_{\Omega(t)} f(x-x',y-y') p(x',y') - \sigma_{\varepsilon}(x',y') dx' dy'$$

In the above equations p is fluid pressure inside the crack, σ_c is the local minimum in-situ stress, w is crack width, C is non-local kernel function which contains all data about layered elastic medium, and f is elastic Influence Function [1,2]. It is assumed that the crack occupies the region denoted by $\Omega(t)$ at time t.

The governing equation of fluid flow inside the crack is as:

$$\delta w/\delta t = \nabla [D(w)(\nabla p - \rho g)] + \delta(x,y)Q,$$

Where p is fluid pressure, ρ is the fluid density, g is the gravity vector, $\delta(.)$ is the Dirac delta function, Q(x,y,t) is the source injection rate, $D(w)=w^3/12\mu$, w is the fracture width and μ is the Newtonian fluid viscosity.[1,2]

3-Numerical analysis

The finite element method was used to calculate the stress intensity factor due to the complexity of the geometry and boundary conditions. The FE model ANSYS was used to simulate the hydraulic fracture process.

In the ANSYS software, there are 3 main ways to evaluate fracture mechanic parameters:

1: Stress intensity factor (K) 2: J-integral JINT (J) 3: Energy release rate VCCT (G)

3.1-Stress Intensity Factors (SIF)

The stress-intensity factor, K, is a parameter to characterize "the stress field ahead of a sharp crack in a test specimen or a structural member". The parameter, K, is related to the nominal stress level (σ) in the structural member and the size of the crack, and has units of (MPa.mm^{0.5}). In general, the relationship is represented by:

$$K = \sigma \sqrt{a p}$$

where *p* is a geometrical parameter that depends on the structural member and crack, *a* is the crack length. All structural members or test samples that have flaws can be loaded to different levels of K. This is similar to the situation where unflawed structural can be loaded to different levels of stress (σ). [3]

3.2- *J*-integral JINT (*J*)

The *J*-Integral evaluation is based on the domain integral method proposed by Shih. The domain integration formulation applies area integration for 2-D problems and volume integration for 3-D problems. Area and volume integrals offer much better accuracy than contour integral and surface integrals, and are much easier to implement numerically [4].

3.3- Energy release rate VCCT (G)

Energy release rate is based on the assumption that the energy needed to separate a surface is the same as the energy needed to close the same surface. The approach for evaluating the energy-release rate is based on the virtual crack-closure technique (VCCT) [5].

4-Numerical simulation

The behaviour of a horizontal well with a crack initiated in its wall was studied under hydrostatic pressure. Figure 1 shows the stress intensity factor for the Modes I fracture versus the crack length. The results indicates that stress intensity factor increases by increasing the crack length. Figure 2 shows the variation of leakage rate with crack length. It can be seen that the leakage rate increases with increasing the crack length.



Figure1: Variation of stress intensity factor with crack length



Figure2: Variation of leakage rate with crack length

5-Conclusion

The goal of this investigation was to better understand the link between crack lengths, leakage rate and stress intensity factor in a typical well.

The linear-elastic fracture toughness of a material with a thin crack was determined from the stress intensity factor (K).

The obtained results indicate that stress intensity factor increases by increase of crack length.

The numerical results indicate that the fluid leakage rate increases by increasing the crack length.

The leak rate analysis provided reasonable results when compared against the available research and test results.

It can be concluded that FE modelling is an efficient tool for the analysis of leakage and stress intensity factor in crack propagation.

Refrences

[1] Adachi J., Siebrits E., Peirce A., and Desroches J., Computer simulation of hydraulic fractures, Int. J. Rock Mech. Min. Sci., 44, pp. 739–757, 2007.

[2] Advani, S.H., Lee, T.S. and Lee, J.K., Three dimensional modelling of hydraulic fractures in layered media: Finite element formulations, J. Energy Res. Tech., 112, 1-18, 1990.

[3] Barsom, John M. and Rolfe, Stanley T., Fracture and Fatigue Control in Structures: Application of Fracture Mechanics, Philadelphia, 1999.

[4]Shih, C. F., B. Moran, and T. Nakamura, Energy Release Rate Along a Three-Dimensional Crack Front in a Thermally Stressed Body, International Journal of Fracture, 30, pp 79-102, 1986

[5] Delorenzi HG, On the energy release rate and the J-integral for 3-D crack configurations, Int. J. Fract. 19, 183–193, 1982.