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VIBRATION ANALYSIS OF DAMAGED PLATES BY HYBRID EXACT STIFFNESS AND FINITE ELEMENT METHOD

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ABSTRACT

During preliminary aircraft design when many alternative configurations and load cases need to be considered, fast and reliable analysis tools are required. The specialist software VICONOPT is designed for efficient, accurate buckling and vibration analysis and optimum design of plates and panels. VICONOPT avoids the modelling and computational costs of finite element analysis by employing the exact stiffness analysis and the Wittrick-Williams algorithm. The vibration behaviour of isotropic plates containing through-the-length damages of different sizes and severities is studied using a novel hybrid method. This method, based on a concept that allows VICONOPT to work along with finite elements to improve the ability of VICONOPT to model cases of damaged plates, uses VICONOPT to model the undamaged parts of the structure in combination with finite elements in the area of the damage. The results show the ability of the hybrid method to handle through-the-length damages efficiently. This achievement opens the door for a wide range of more complicated damages that can be studied using this method.

Keywords: *Vibration, isotropic, damages, exact stiffness, finite element*

1. Introduction

Minimizing the mass of an aircraft's structure reduces the cost of materials and manufacturing, as well as fuel consumption and atmospheric emissions. During the preliminary design when many alternative configurations and load cases need to be considered, fast and reliable analysis tools are required. The VICONOPT (VIpasa with CONstraints and OPTimization) program is designed for efficient, accurate buckling and vibration analysis and optimum design of plates and stiffened panels, and is typically about 10 times faster than the equivalent finite element (FE) analysis [1]. This program is based on the exact finite strip method which assumes a continuous distribution of stiffness over the structure. VICONOPT is able to model plates which have different properties in the transverse (y) direction and even different thickness by connecting different plates in this direction. However, all the plates need to have the same length in the longitudinal (x) direction.

VICONOPT is able to model many different cases of damaged plates as long as the damaged region stretches along the whole length of the plate in the x direction. Damghani et al. [2] studied the critical buckling of composite plates with through-the-length delaminations which satisfy these prismatic requirements. This work was extended (Damghani et al. [3]) to study the global buckling behaviour of a composite plate with a single rectangular delamination. This method is based on replacing the longitudinal portion of the plate containing the delamination with an equivalent prismatic structure, which is capable of representing the global buckling behaviour but cannot capture local effects.

The combined VICONOPT and FE method presented here, denoted VFM, is based on using the FE method to model the longitudinal portion of the plate containing the damage as shown in Fig 1. However, the FE method has been merged with VICONOPT process to model the whole plate. This work extends the capability of VICONOPT to use computationally efficient analysis for different cases of plate damage which could not previously be modelled.

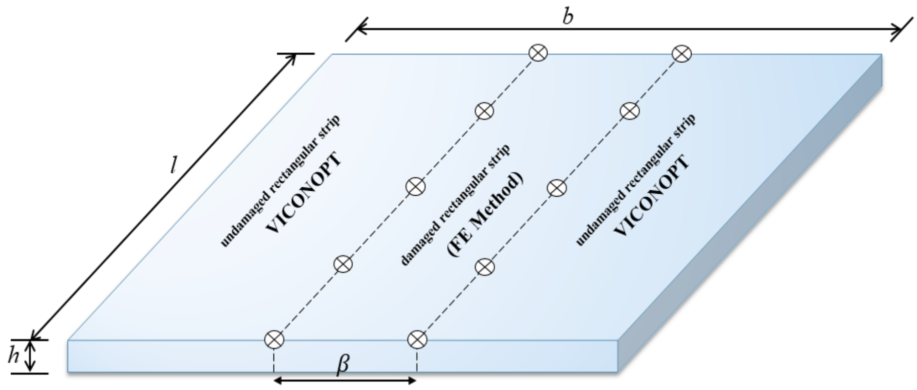

Figure 1: VICONOPT and FE method modelling damaged plate

2. Problem definition

During the early stages of design when many alternative configuration and load cases are considered, fast and reliable analysis tools are required. Finite element analysis (FEA) is capable of handling many combinations of loading and boundary conditions for different cases of damage but it is computationally expensive. These high computational costs are avoided in VICONOPT by employing exact strip solutions and the Wittrick-Williams algorithm [4]. However due to its prismatic requirements, VICONOPT can model damaged plates directly only if the damaged region stretches along the whole length of the plate. The aim of this study is to develop a novel method which can be used to improve the ability of VICONOPT to model more complex cases of damaged plates. For validation purposes and to demonstrate the efficiency of the approach a comparison is first made with FEA for an isotropic plate with through-the-length damage.

3. Theory and Formulation used in VFM

VICONOPT incorporates two earlier programs, VIPASA and VICON. VIPASA provides a powerful analysis for vibration and buckling of prismatic plate assemblies with simply supported ends. However, if the structure is under in-plane shear loading, the mode shapes will be skewed and the end conditions will not be satisfied. Thus the applicability of VIPASA is limited. VICON [5] provides a solution to this problem by coupling the VIPASA stiffness matrices for different wavelength responses through the Lagrangian Multiplier Method [5]. The complete VIPASA generality and capability are thus retained in the VICON program, which satisfies the end conditions through point constraints and also permits attachments to a supporting structure [6]. Thus the VICON stiffness matrix comprises a series of VIPASA stiffness matrices which are coupled by the constraints. It is complex valued and can be expressed as

$$\mathbf{K}_{VICON} = \begin{bmatrix} \mathbf{K}_{Global\,VIPASA} & \mathbf{C}^{\boldsymbol{H}} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \tag{1}$$

where $\mathbf{K}_{Global\,VIPASA}$ is the global VIPASA stiffness matrix, i.e. a series of uncoupled VIPASA matrices at different half-wavelengths of response, $\mathbf{C}$ is the global plate assembly constraint matrix, and $\mathbf{C}^H$ is its Hermitian transpose.

The method proposed in this paper uses the VICONOPT solution procedure for finding the critical buckling loads and natural frequencies for damaged plates, by using a combination of VICONOPT and FE stiffnesses as shown in Fig. 1. Embedded damage is modelled by including finite elements with different stiffness properties within the damaged strip. VICONOPT then assembles the hybrid stiffness matrix of the plate by using Lagrangian Multipliers to couple the VICON and FE components, as follows.

$$\mathbf{K}_{Global} = \begin{bmatrix} \mathbf{K}_{\ Global\,VIPASA} & \mathbf{0} & \boldsymbol{C}_1{}^{H} \\ \mathbf{0} & \mathbf{K}_{FE} & \boldsymbol{C}_2{}^{T} \\ \boldsymbol{C}_1 & \boldsymbol{C}_2 & \mathbf{0} \end{bmatrix} \tag{2}$$

Here, the constraint matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ enforce equal displacements and rotations at the nodes connecting the undamaged and damaged strips. $\mathbf{C}_1$ also includes any end support conditions in the undamaged regions. $\mathbf{C}_2{}^T$ is the transpose of $\mathbf{C}_2$. $\mathbf{K}_{FE}$ is the FE stiffness matrix for the damaged rectangular strip. In the case of vibration problems $\mathbf{K}_{FE}$ takes the form

$$\mathbf{K}_{FE} = \mathbf{k} - n^2\mathbf{m} \tag{3}$$

where n is the frequency, $\mathbf{k}$ and $\mathbf{m}$ are the static stiffness matrix and equivalent mass matrix of the damaged rectangular strip.

4. Numerical results

In this section, plates with through-the-length damage are analysed using VFM, and also pure FEA using ABAQUS/Standard [7]. The rectangular mesh is constructed using an approximate global size of 5mm, homogeneous continuum shell and elements and a linear perturbation procedure is used. The plates are simply supported on all four sides. Natural frequencies are determined for plates containing different sizes and severities of through-the-length damage. Plates are assumed to be isotropic for simplicity, in order to verify the concept of combining VICONOPT and FE analysis. The isotropic plates examined have length l=100mm, width b=100mm, and thickness h=1mm. They have material properties of Young's modulus E=110 kN.mm^{-2} and Poisson's ratio ν=0.3. Fig 1 shows a plate containing a centrally located through-the-length degradation of stiffness over a width β. In this parametric study, the width of the damage β is varied and three values of stiffness reduction factor for the damaged region are considered: f=0.75, f=0.50 and f=0.25.

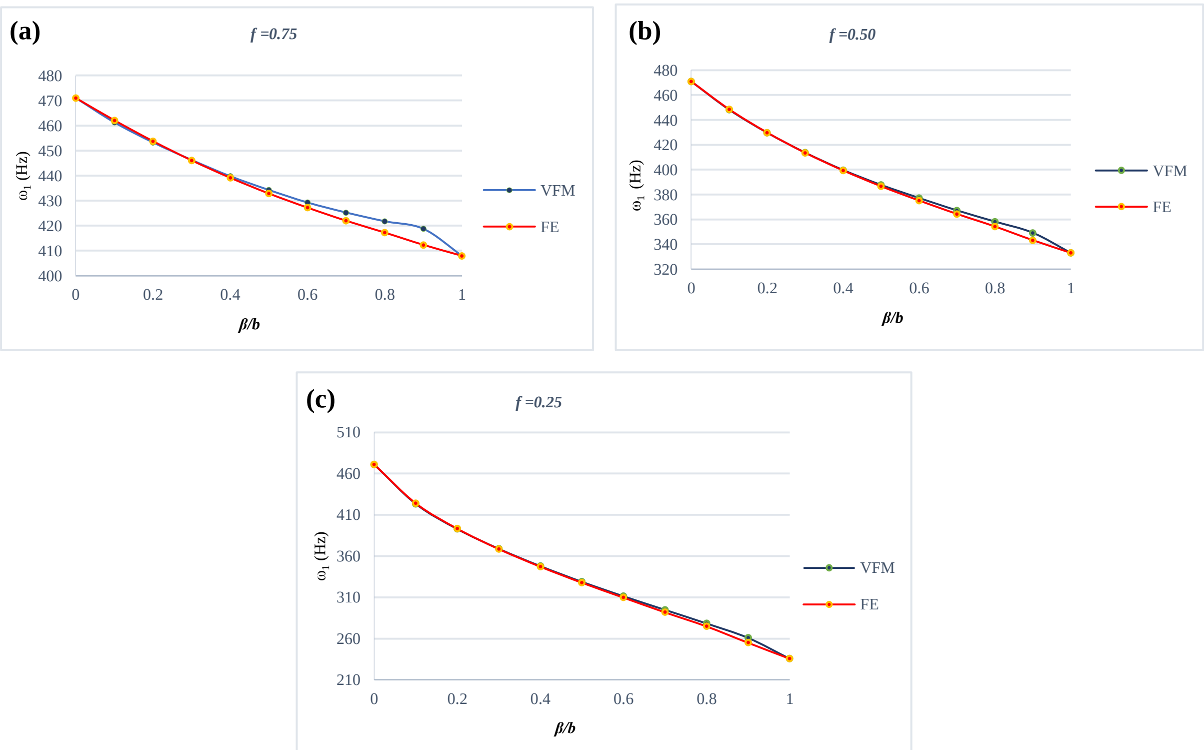

Figure 2: Plots of lowest natural frequency (ω_1) against width β of centrally located through-the-length damage, for different values of stiffness reduction factor f. (a) f =0.75, (b) f =0.50, (c) f =0.25.

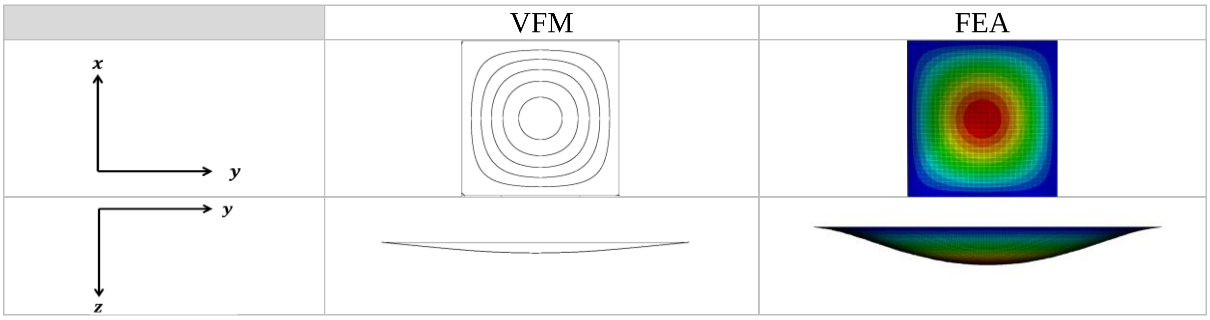

Figure 3: VFM and ABAQUS plots of lowest natural frequency mode for plate with through-the-length damage of β/b=0.2

Fig. 2 shows that there is an excellent agreement between the two methods when $0 \leq \beta$/b ≤ 0.7. There is a negligible difference (maximum 2%) between the natural frequencies obtained from FEA (Abaqus) and VFM when $0.7 < \beta$/b < 1.0. Fig. 3 illustrates the vibration mode shapes obtained from VFM and FEA.

5. Conclusions and future work

VFM is based on a concept that allows the computationally efficient program VICONOPT to work along with the finite element method to improve its ability to model cases of damaged plates. This method has been used for the natural frequency analysis of plates containing through-the-length damage. The analysis results are validated against pure FEA. This novel method will be used for more complicated cases of damage that VICONOPT could not previously model directly, such as embedded rectangular damage.

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