Proceedings of the 24th UK Conference of the
Association for Computational Mechanics in Engineering
31 March - 01 April 2016, Cardiff University, Cardiff

An approach for dynamic analysis of stationary cracks using XFEM

***J. Navarro-Zafra[1], J. L. Curiel-Sosa[1], M. C. Serna Moreno[2], C. Pinna[1], J. L. Martínez Vicente [2], N. Rohaizat [1], B. Tafazzolimoghaddam[1]**

[1] Department of Mechanical Engineering, University of Sheffield. Sir Frederik Mappin Building, Mappin Street, Sheffield, S1 3JD, UK
[2] E.T.S. de Ingenieros Industriales de Ciudad Real, Universidad de Castilla La Mancha, Edificio Politécnico. Avda. Camilo José Cela s/n, Campus Universitario, 13071, Ciudad Real (Spain)

*jnavarrozafra1@sheffield.ac.uk

ABSTRACT

A numerical implementation of the eXtended Finite Element Method (XFEM) is presented. The proposed approach solves the system of discrete equations using an explicit integration scheme and it is capable of addressing dynamic and static fracture mechanics problems. Special attention to the mass matrix construction is required in order to avoid instability issues such a null stable time increment. Hence, different mass lumping strategies are adopted for enriched elements. The in-house implementation of this approach, so-called X^2FEM, is embedded in the main in-house FE platform called MULE. Numerical tests demonstrate that the proposed approach is able to provide an accurate calculation of static and dynamic stress intensity factors (SIFs) for different geometries and loading scenarios. Finally, in order to extend our point of view, an experimental analysis of a 10° off-axis carbon fibre laminate is carried out using Digital Image Correlation (DIC).

Key Words: *Extended finite element Method (XFEM); Explicit time integration; Stress intensity factors; Digital image correlation*

1. Introduction

During the last years, the XFEM [1] has been used successfully in the simulation of moving cracks throughout a structure such as [2, 3, 4]. Using this method for the simulation of fracture, the mesh is not updated at each time step in opposition to the Finite Element Method (FEM), where the mesh must be updated at each time step in order to conform with the moving discontinuity. Although, the FEM has solved many problems of interest such as [5, 6, 7], XFEM is computationally more efficient than the FEM while simulating the evolution of cracks.
Based on previous experience with XFEM [8] and FE modelling of damage [9] using commercial codes, the objective of this work is to present a numerical approach to analyze stationary cracks in the framework of XFEM. This approach will improve the flexibility during programming since an entire in-house code, so-called X^2FEM, is available. This in-house code is implemented in the main FE platform called MULE. The approach proposed considers an explicit time integration scheme. In this case, the well-known central difference method (CDM) is adopted for time discretisation. A diagonal mass matrix is used for solving the discrete momentum equation. Hence, the part of the mass matrix corresponding with the standard Degree Of Freedom (DOF) are lumped by direct mass lumping. However, for the lumping of the enriched DOF there is not straightforward way and a limitation exists. The limitation was found by Belytschko et al. [10]. Basically, they found out that the critical time step of the explicit XFEM decreases notably as a discontinuity gets closer to nodes. To alleviate this restriction, Belytschko et al. [10] used an implicit integrator for the enriched element and explicit integrator for standard elements. Recently, other possible solutions for solving this limitation are based on using mass lumping strategies. In this work, using specific lumping techniques for enriched elements the diagonalized mass matrix is obtained avoiding the possibility of having a null critical time step.

2. General problem

A two-dimensional dynamic problem is considered where a body Ω with boundary $\partial\Omega$ is defined. This boundary is divided into $\partial\Omega_u$, $\partial\Omega_F$ and Γ_c (see Figure 1). Hence, $\partial\Omega = \partial\Omega_u \cup \partial\Omega_F \cup \Gamma_c$, where $\partial\Omega_u$ represents the prescribed displacements in the body Ω, $\partial\Omega_F$ is the part of the body subjected to surface forces and Γ_c corresponds to the displacement discontinuity e.g. a crack. Note that crack 's faces are traction free. The motion of the body is defined by the displacement $\boldsymbol{u}(\boldsymbol{x}, t)$, which is a function of the location of the material point $\boldsymbol{x}$ and the time t. The material has a linear behaviour, it is isotropic and its mass density is ρ. The body presents applied displacements $\bar{\boldsymbol{u}}$ on the Dirichlet boundary $\partial\Omega_u$ and applied traction $\bar{\boldsymbol{t}}$ on the Neumann boundary $\partial\Omega_F$; $\partial\Omega_u \cap \partial\Omega_F = \emptyset$, $\partial\Omega_u \cap \Gamma_c = \emptyset$, $\partial\Omega_F \cap \Gamma_c = \emptyset$. The outward normal vector in the material boundary is defined as $\boldsymbol{n}^\perp$ and $\boldsymbol{b}$ is the body force per unit mass. Thus, the strong form of the problem is written as follows:

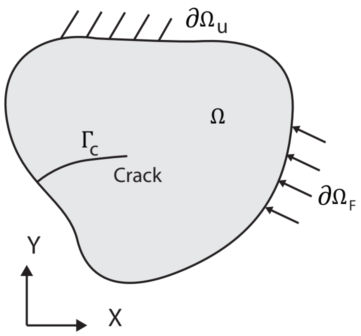

Figure 1: A two dimensional body cracked and its boundaries.

$$\nabla\boldsymbol{\sigma} + \rho\boldsymbol{b} - \rho\ddot{\boldsymbol{u}} = 0 \; in \; \Omega \tag{1}$$

subjected to the boundary conditions:

$$\boldsymbol{u} = \bar{\boldsymbol{u}} \; on \; \partial\Omega_u \tag{2}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n}^\perp = \bar{\boldsymbol{t}} \; on \; \partial\Omega_F \tag{3}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n}^\perp = 0 \; on \; \Gamma_c \tag{4}$$

The constitutive relation of the material is written:

$$\boldsymbol{\sigma}(\boldsymbol{x}, t) = \boldsymbol{C} \cdot \boldsymbol{\epsilon}(\boldsymbol{u}(\boldsymbol{x}, t)) \tag{5}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{C}$ the constitutive matrix and $\boldsymbol{\epsilon}$ the strain tensor.

3. Numerical framework

The displacement at a generic point $\boldsymbol{x}$, $\boldsymbol{u}(\boldsymbol{x}, t)$ is approximated by $\boldsymbol{u}^h$ using continuous and discontinuous terms as follows [11]:

$$\boldsymbol{u}^h(\boldsymbol{x}, t) = \boldsymbol{u}^{cont}(\boldsymbol{x}, t) + \boldsymbol{u}^{cut}(\boldsymbol{x}, t) + \boldsymbol{u}^{tip}(\boldsymbol{x}, t) \tag{6}$$

where $\boldsymbol{u}^{cont}$ corresponds to the continuous approximation of the displacement, $\boldsymbol{u}^{cut}$ corresponds to the discontinuous approximation for addressing the crack and $\boldsymbol{u}^{tip}$ denotes the discontinuous approximation corresponding to the crack tip.

For the implementation of approach presented, an in-house code developed in MATLAB is built. The element type considered in this code is a 4-node quadrilateral element. Each node for a standard element presents two DOF. The nodes of full cracked elements have two classical DOF and two additional DOF addressing the strong displacement jump. The construction of the global mass matrix is not a trivial task. The mass lumping strategy adopted for the enriched part is critical for the stability of the CDM. Therefore, different mass lumping strategies are adopted depending on the type of enrichment. For standard elements, the calculation of the stiffness matrix considers four integration points. In the other hand, for elements containing the tip of the crack or cut by the crack, the integrals for the calculation of internal forces cannot be derived by standard quadrature methods since the integrand is defined as discontinuous. Therefore, the standard Gauss quadrature does not adequately considered the discontinuity and a subdivision of the elements into triangles is required. This subdivision of the elements has just integration purposes, hence, no additional DOF are added to the system.

4. Computational tests

In order to check the performance of the proposed approach, a semi-infinite stationary crack within an infinite plate is simulated. The plate is loaded in its vertical edge as depicted in Figure 2. The influence of the external loading on the Stress Intensity Factor (SIF) calculation is addressed. Hence, the load applied is introduced in three different manners: as a step, a ramp and a sinusoidal wave. The dynamic SIFs obtained from the code are compared with the analytical solution.

4.1. Analysis of a stationary crack: mode I

A schematic representation of the problem under consideration is depicted in Figure 2 where a tensile stress is applied perpendicular to the crack. The analytical solution of the SIF in mode I (denoted as K_I) for a linear elastic material was first proposed by Freund [12]. The analytical solution was obtained under the assumptions of an infinite plate with a semi-infinite crack. This solution is valid till the tensile wave stress is reflected in the bottom of the plate and reaches again the crack tip. The dimensions of the plate are L=10 m, h=2 m and the crack length is a=5 m. The material properties are presented on Table 1 where E is the elasticity modulus, ν the poisson ratio and ρ the density.

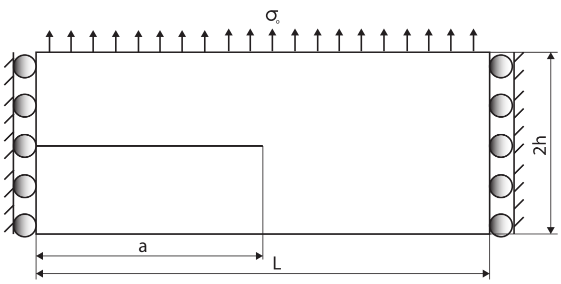

Figure 2: Geometry and loading for a infinite plate with a semi-infinite crack.

Table 1: Mechanical properties

$E[GPa]$	ν	$\rho[kg/m^3]$
210	0.3	8000

In Figure 4, the normalized mode I dynamic SIF $\frac{K_I}{\sigma_o\sqrt{h}}$ is depicted against the normalized time $\frac{t}{t_c}$ being $t_c = \frac{h}{c_d} = 3.36 \cdot 10^{-4}s$, where c_d is the dilatational wave speed. This graph serves to point out the mesh independence of the solution using two different discretisations with 92x39 and 140x59 elements. These discretisations were considered in order to have an aspect ratio of approximately one within the mesh. Note that the time step considered is $\Delta t_c^{XFEM} = 5\mu s$ (a simulation of 200 time steps). The tensile load applied $\sigma_0(t)$ is defined as $\sigma_0(t) = \sigma g^n(t)$, where $\sigma = 500MPa$ and $g^n(t)$ defines the way that the load is provided. Initially, a step load $g^1(t)$ is applied to the plate as follows:

$$g^1(t) = \begin{cases} 0 & \text{if} \quad t \leq 0 \\ 1 & \text{otherwise} \end{cases} \tag{7}$$

Two more loading scenarios were considered: a ramp and a sinusoidal wave. The results depicted on Figure 4 while loading with $g^1(t)$ show a reasonable agreement between the analytical and the computational solution for the coarse and fine meshes. Consequently this proves the ability of the proposed code for the calculation of SIF considering different discretisations.

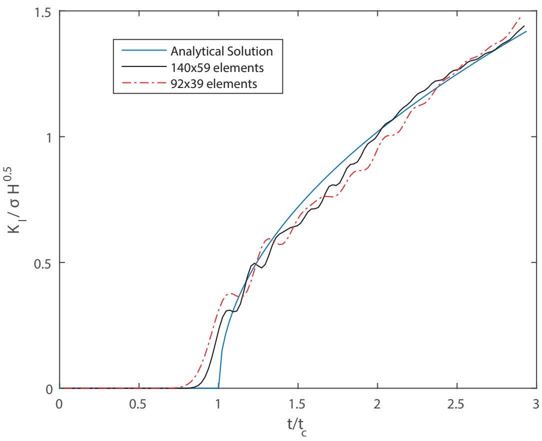

Figure 3: Normalized mode I stress intensity factor against normalized time for a stationary semi-infinite crack. The analytical solution is plotted as well as the computational solution considering two different discretizations: 92 by 39 and 140 by 59 elements .

5. Validation using Digital Image Correlation

Digital Image Correlation (DIC) [13][14] is an optical technique based on digital image processing and numerical computing. This technique provides the full-field displacements and strains for a surface by comparing a digital image reference (un-deformed) with a deformed image stage.

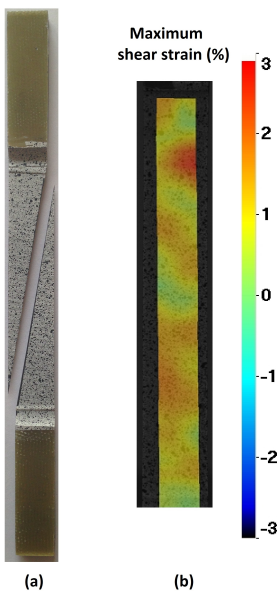

Figure 4: (a) Macro-crack observed during experimental tensile testing and (b) Map of maximum shear strain before failure.

In order to extend our point of view, a 2D experimental analysis of a 10° off-axis carbon fibre laminate is carried out using Digital Image Correlation (DIC). Therefore, the full-field of strains was obtained and those outcomes were compared with simulations. In Figure 5 (a), it is depicted the crack path for the 10° laminate once the specimen breaks. Additionally, in Figure 5 (b), it is presented a map of maximum shear strain before the macro-crack appears. Notice a peak of maximum shear strain in the zone where the crack is initiated.

6. Conclusions

A numerical approach for the simulation of stationary cracks in the context of XFEM is presented. This approach is numerically implemented in an in-house code programmed in MATLAB. By means of several tests, the performance of the in-house code is tested. The results obtained and its comparison with the analytical solutions proves the reliability of the approach. Additionally, in order to expand our point of view, an experimental analysis is carried out using DIC.

Acknowledgements

This work has been financially supported by the Engineering and Physical Sciences Research Council (EPSRC) through grant reference EP/K503149/1. Additionally, the first author would like to thank the University of Sheffield Learned Societies fund for covering all expenses related to this conference.

References

[1] Belytschko, T. and Black, T., Elastic crack growth in finite elements with minimal remeshing. *International journal for numerical methods in engineering*, 45(5), 601–620, 1999.

[2] Curiel-Sosa, J.L. and Karapurath, N., Delamination modelling of GLARE using the extended finite element method. *Composites Science and Technology*, 72(7), 788-791, 2012.

[3] Ibbett, J. , Tafazzolimoghaddam, B. , Hernandez Delgadillo, H. , Curiel-Sosa, J.L., What triggers a microcrack in printed engineering parts produced by selective laser sintering on the first place?. *Materials and Design*, 88, 588-597, 2015.

[4] Serna Moreno, M.C. , Curiel-Sosa, J.L. , Navarro-Zafra, J. , Martínez Vicente, J.L. and López Cela, J.J. , Crack propagation in a chopped glass-reinforced composite under biaxial testing by means of XFEM. *Composite Structures*, 119, 264-271, 2014.

[5] Curiel-Sosa, J.L. , de Souza Neto, E. , Owen, D.R.J. , A combined implicit-explicit algorithm in time for non-linear finite element analysis. *Communications in Numerical Methods in Engineering*, 22(1), 63-75, 2006.

[6] Curiel-Sosa, J.L. , Beg, O.A. , Murillo, J.M.L. , Finite element analysis of structural instability using an implicit/explicit switching technique. *International Journal of Computational Methods in Engineering Science and Mechanics*, 14(5), 452-464, 2013.

[7] Curiel-Sosa, J.L. and Gil, A.J., Analysis of a continuum-based beam element in the framework of explicit-FEM. *Finite Elements in Analysis and Design*, 45(8-9), 583-591, 2009.

[8] Navarro-Zafra, J. , Curiel-Sosa, J.L. , Serna Moreno, M.C. ,Three-Dimensional Static and Dynamic Analysis of a Composite Cruciform Structure Subjected to Biaxial Loading: A Discontinuum Approach. *Applied Composite Materials*, Article in Press, 2015.

[9] Navarro-Zafra, J. , Curiel-Sosa, J.L. and Serna Moreno, M.C. ,Mixed-mode damage into a CGRP cruciform subjected to biaxial loading. *Composite Structures*, 133, 1093-1100, 2015.

[10] Belytschko, T. , Chen, H. and Xu, J. and Zi, G. Dynamic crack propagation based on loss of hyperbolicity and a new discontinuous enrichment. *International Journal for Numerical Methods in Engineering*, 58(12), 1873-1905, 2003.

[11] Moes, N. , Dolbow, J. , Belytschko, T. .A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering*, 46(1), 131-150, 1999.

[12] Freund, L. B. .Dynamic fracture mechanics. *Cambridge university press*, 1998.

[13] Chu, T.C. , Ranson, W.F. , Sutton, M.A. .Applications of digital-image-correlation techniques to experimental mechanics. *Experimental Mechanics*, 25(3), 232-244, 1985.

[14] Caminero, M.A. , Pavlopoulou, S. , Lopez-Pedrosa, M. , Nicolaisson, B.G. , Pinna, C. , Soutis, C., Sutton, M.A. .Analysis of adhesively bonded repairs in composites: Damage detection and prognosis. *Composite Structures*, 95, 500-517, 2013.